The Underwriting Profit Provision

1. Introduction
   a. Profit Provisions & Actual Underwriting Profit
      i. Provision for underwriting profit plus sum of provisions for losses and expenses equals the total premium rate
      ii. Loss provision is assumed to include adequate load for catastrophes
      iii. Underwriting profit provision may differ from the actual underwriting profit
   b. Historical Overview
      i. 1921 – 1960’s, a 5% underwriting profit was common along most lines
      ii. 1970 – 1980’s, impact of investment income & taxes on the bottom line became increasingly important
      iii. Rising price of insurance made affordability an issue
         1. Led to changes in the calculation of the underwriting profit provision
         2. Often intended to lower the price by lowering the rates
      iv. We now face a variety of algorithms of computing the provision
         1. In some states, investment income percentage is subtracted from a more traditional provision to get the final profit provision
         2. In others, the provision is set to theoretically give the company a “proper” total rate of return, as determined by regulators
   c. Underwriting Profit & Total Profit
      i. Total profit = sum of underwriting gains plus investment gains, less income taxes
      ii. Investment Gains:
         1. Interest
         2. Dividends
         3. Real Estate Income
         4. Realized Capital Gains
      iii. Company can make an overall profit even if u/w profit is negative
      iv. Negative u/w profit can reduce income taxes
      v. One might theoretically want to define an underwriting profit provision as adequate if it would lead to an adequate total profit net of investment income and income taxes, but it’s not obvious how to apply this to ratemaking.
         1. Ratemaking is done on a prospective policy year basis
         2. Total return is commonly measured on a calendar year basis
            a. GAAP ROE is the usual measure of return
            b. The ratio of calendar year GAAP income over GAAP equity
            c. Partly dependent on the past
         3. Rates are made looking to the future, and rates made today will influence earnings in many subsequent calendar years
      vi. A different approach is to set rates with an explicit offset for investment income on policyholder supplied funds
         1. These funds are the accumulated balance of paid premiums less the sum of loss and expense payments
         2. Offset could be estimated by projecting underwriting cash flows prospectively or by looking at calendar year data
            a. Must estimate what portion of the total investment income is generated by policyholder supplied funds
            b. Remaining portion is earned on stockholder supplied funds
               i. The insurance company’s surplus
               ii. Amounts needed to offset the portion of the loss and expense reserves not covered by premium
               c. Conceptual split between the funds helps in gauging adequacy of a total return
            d. Positive total return doesn’t imply a fair return for stockholders
   d. Five Types of Underwriting Profit
      i. Underwriting Profit Provisions included in manual rates and in rate filings to change manual rates
      ii. Corporate target underwriting profit provisions
         1. Used internally to decide how much to charge
         2. Presumably sufficient to yield an expected return comparable to the market yield
      iii. Breakeven underwriting profit provisions
         1. Generate an expected return to stockholders equal to the return on risk free investments
         2. Ignores the risk-return tradeoff
iv. Charged Underwriting Profit Provision: obtained by applying schedule rating and experience rating modifications to the manual rate

v. Actual Underwriting profits
   1. Differ from those charged because provisions for losses and expenses will rarely be exactly accurate
   2. If actual results are consistently below those charged, check the methods used to estimate losses & expenses for bias.

e. Theories of Regulation
   i. “Rate of Return Regulation” Approach
      1. Stringently regulate premium rates so that companies should be able to anticipate adequate total returns
      2. Justified by arguing that insurance company rates should be regulated in a fashion similar to the way that utility rates are regulated

   ii. Constrained Free Market Theory
      Assumptions:
      a. Manual rate is reasonably adequate
      b. There is some limited flexibility to price insurance
      c. Competitive market should force the price of insurance to the optimal level
      d. Manual rates are a “starting point”

   iii. Large number of firms and the relatively low market share of even the biggest insurers are cited as evidence that a competitive market model is the appropriate one.

f. Summary of Different Models
   i. Calendar Year Investment Offset Procedure
      1. Reduce Traditional Profit load by an offset
      2. Offset is an investment figured derived from calendar year data

   ii. Present Value Offset Approach
      1. Reduce traditional profit load by an offset
      2. Offset reflects the differences in the present values of the loss payment patterns of the line of business under review and some short-tailed reference line

   iii. Calendar Year Return Method
      1. Profit Provision is set to achieve a selected target return
      2. Return is calculated with investment income figures derived from calendar year data

   iv. Present Value of Income over the Present Value of Equity
      1. Present value return is defined as the present value of accounting income over annualized present value of equity
      2. Adjusts underwriting profit until the present value return is equal to a target return

   v. Present Value Return on Cash Flow Model
      1. Provision is set so that the present value of the underwriting cash flows and investment income (less taxes) is equal to the present value of the changes in equity
      2. Underwriting cash flows and investment income (less taxes) are discounted using the rate of return on investments
      3. Changes in equity are discounted at the target rate of return

   vi. Risk-Adjusted Discounted Cash Flow Method
      1. Premium is calculated so that the present value of the premium payments will equal the present value of paid losses, expenses and taxes
      2. Since losses are not known, they are discounted at a new money risk-adjusted rate, which is lower than the new money risk-free rate

   vii. Internal Rate of Return on Equity Flows
      1. Estimates flows of money between a hypothetical stockholder and a hypothetical company
      2. “Equity Flows” are related to the projected stream of total income and the assumed surplus requirements
      3. Profit provision is found by adjusting the premium until the yield on the equity is equal to a target return

   viii. Notes:
      1. No simple empirical test of which model is better
      2. Judgment on the adequacy of actual profits depends on some underlying economic theory
      3. A model that accurate estimates what underwriting profits were doesn’t necessarily way what they should have been.
2. **Underwriting Profit Provision in the Premium Formula**

\[
P = (1 + c)L + FX + P \cdot VR + P \cdot U = \frac{(1 + c)L + FX}{1 - (VR + U)}
\]

\[
CR = VR + \frac{(1 + c)L + FX}{P}
\]

\[
U = 1 - CR
\]

3. **Calendar Year Investment Income Offset Method**

An offset is made against the “traditional” underwriting profit provision in order to arrive at a final number.

The first step is to find the ratio of the pre-tax investment income relative to average invested assets. Then, applying appropriate tax rates, we can arrive at an after-tax portfolio yield.

Once this is calculated, we need to relate invested assets to premiums. To do this, we need to estimate the amount of the policyholder-supplied funds. This is done indirectly, by looking at both the unearned premium reserve and the loss reserve.

Looking first at the unearned premium reserve, the first step is to obtain the ratio of average direct unearned premium to direct earned premiums, then reduce for prepaid expenses and premium balances due. We reduce for prepaid expenses because the company has already paid them out, so the premiums paid to cover them are not generating investment income. Regarding the premium balances, the company doesn’t have the cash to invest, it’s merely an IOU. Unpaid premium balances are obtained from the Annual Statement, ratioed against direct premium, and then the ratio is subtracted from the previously calculated unearned premium ratio net of prepaid expense.

Looking at the loss reserve, we compute the ratio of calendar year loss reserves to calendar year incurred losses. For stability, we might use a multi-year average for this. Then, multiply the permissible loss ratio to arrive at a ratio of reserves to premiums. A reserves-to-premium ratio consistent with the permissible loss ratio in the prospective rate is calculated as the product as the permissible loss ratio times the historic average reserves-to-incurred loss ratio. This provides the policyholder credit for the investment income associated with the loss provision of the premium

\[
PHSF = \left( \frac{UEPR}{PREM} \right) (1 - PPACQ) - \frac{RECV}{PREM} + PLR \cdot \left( \frac{LRES}{INCL} \right)
\]

\[
U = U^0 - i_{AFIT} \cdot PHSF
\]

The permissible ratio is used to calculate the profit provision, which is used to calculate the permissible loss ratio. The calculation should be done iteratively until the permissible loss ratio is consistent with the offset.

Advantages:
- Practicality
  - Figures come from documents already filed (annual statements)
  - Numbers are generally stable
- Calculation is short
- Logic is not too difficult

Disadvantages
- Lack of an underlying general economic theory to support the calculation
- Possibility of distortion when there’s rapid growth or decline in volume or changes in loss reserve adequacy

Conclusion:
- Algorithm accounts for investment income in a fairly straightforward manner
- In stable growth scenarios with stable reserving patterns, it is appropriate for use in rate filings.
4. Present Value Offset Method

This method also offsets the traditional underwriting profit provision. In this case, the offset is based on the difference between the present value of losses for a short-tailed reference line and the present value of losses for the line under review.

Let $x(j)$ be the fraction of ultimate losses paid out at the end of the $j^{th}$ year. The present value of losses:

$$ PV(L, i) = L * PV(x, i) = L * \sum_{j=1}^{n} x(j) v^j $$

If $x^0$ is a reference loss payout pattern and $x$ is the loss payout pattern for the line under review, letting the total amount of loss be given by a permissible loss ratio and expressing the present value loss differential as a ratio to premium, we obtain:

$$ DELPVLRL = PLR \left( PV(x^0; i) - PV(x; i) \right) $$

Then, the underwriting profit provision: $U = U^0 - DELPVLRL$

The intent is to adjust for the differences in investment income potential between lines. Ignore loss expenses and rewrite the premium formula:

$$ P = PVL + FX + VR * P + U^0 * P + L(1 - PVx^0) $$

PVL = present value of losses for the line

L = Full value of losses for the line

$PVx^0 =$ Present value loss pattern for the reference line

The loss provision is now being counted only at its present value, while the profit provision is the sum of the traditional underwriting profit provision, plus the investment income on the reference line.

Selection of the interest rates for discounting losses is critical. One might prefer new money yields, since the ratemaking is done prospectively, but portfolio yields are more stable and more verifiable. To account for income taxes, we could use an after-tax interest rate for discounting the loss payout patterns. This would only account for taxes on the investment income associated with the present value loss provision. To correct this, a more elaborate model can be constructed within the same general framework by computing projected income taxes for the reference line and the line under review.

Application of the interest rate on an after-tax basis can be done either prospectively or retrospectively. When going prospectively, apply the appropriate prospective tax rates to the pre tax yields, and calculate the after-tax yield by weighting the after tax yields by the assumed mix of assets. Retrospectively, the tax rate is derived from a companies’ actual income tax rate on all income for the prior year. It’s assumed that this is a good estimation of the current year’s taxes. This may not be true for 2 reasons:

1) Tax paid in the past reflects past underwriting profits, which may or may not be consistent with the underwriting profit provision in the prospective rates
2) The use of the prior year’s actual tax rate operates perversely; it penalizes companies that lost money, and rewards those that were profitable.

The method does account for investment income in a direct and simple fashion. It’s not distorted by rapid growth or decline like the previous method, and there’s no need to select a target return or the estimate a surplus requirement

5. Calendar Year Return on Equity Method

This method uses calendar year figures to calculate a rate of return on equity.

$$ ROE = \frac{INC}{EQ} = \frac{(U * P) + II - FIT}{EQ} $$

$ROE =$Return on Equity

$INC =$Total Income

$U =$Underwriting Profit Provision

$P =$Premium

$FIT =$Federal Income Tax

$EQ =$Equity
Equity is set by first choosing a Statutory Surplus requirement. Caap Equity can then be estimated by applying a historic equity to surplus ratio. A more complicated option would be to add in the DAC expense balance and make other adjustments.

It’s useful to split the tax into two components: tax on underwriting income and tax on investment income. The total income numerator may be written as the sum of after-tax investment income and after-tax underwriting income.

\[ INC = (1 - t_u) * U * P + II_{AFIT} \]

After tax income is computed using an after-tax yield calculated in the same way as in the Calendar Year Investment Offset Method. The after-tax yield should be multiplied against all investible funds.

\[ II_{AFIT} = i_{AFIT} (PHSF * P + S) \]

If the target return and necessary leverage ratios are given, then we can solve for the corresponding underwriting profit provision to obtain the formula:

\[ U = \left( \frac{1}{1 - t_u} \right) \left[ r * \left( \frac{QSR}{PSR} \right) - i_{AFIT} \left( PHSF + \frac{1}{PSR} \right) \right] \]

\[ r = \text{Target Return on Equity} \]
\[ PSR = \text{Premium to Surplus Ratio} \]
\[ QSR = \text{Equity to Surplus Ratio} \]

Using a calendar year return method has some issues:
1. Subject to biases due to rapid growth or changing reserve adequacy
2. How to select and defend a target return
3. GAAP Equity or Statutory Surplus denominator
4. How to pick an appropriate leverage ratio

If we can address these issues, the methodology does have positive features:
1. Figures used in the calculation are published, making verification easy
2. Produces a return on equity which is (in some sense) comparable to the GAAP ROE commonly used to measure profitability

6. Present Value of Income over Present Value of Equity

This method sets a profit provision in order to achieve a target “present value return,” which is the ratio of the present value of accounting income over the present value of equity. The model is set up so that income is earned and equity is evaluated in accord with accounting standards.

Main equation = \( r = PV(INC)/PV(EQ) \)

First, construct a model to produce Statutory and GAAP Balance sheets and income statements for a hypothetical company writing a single policy. Assumptions:
- Premium earned evenly over the policy term
- UEPR = written premium – premium earned to date
- L & LAE Expense incurred uniformly over the policy term
- L & LAE reserve = Incurred to date – Paid to date
- Reserve is not always adequate, it implicitly includes IBNR claims as well as case reserves
- Under STAT:
  - Premium Tax Commission Acquisition expense earned incurred up front
  - General expenses need not be incurred at policy inception
  - Expense reserves posted as difference between incurred to date and paid to date
- Under GAAP:
  - Premium tax and commission incurred uniformly
  - Incurrence of other acquisition expenses may be deferred.
  - The difference between STAT incurred to date and GAAP incurred to date is the DAC Balance, and is a GAAP Asset

Investment income for each accounting period is approximated by applying the pre-tax investment yield to the average balance of invested assets during the period. This average balance is calculated by taking total assets and subtracting
non-investible assets (like premiums receivable). Assets are equal to the sum of Statutory Reserves and Statutory Surplus.

Income taxes should be based on relevant provisions of the tax code as applied to the hypothetical company. If the tax accounting income is negative, either allow the negative tax, or only allow the negative tax if it offsets positive taxes paid in previous periods.

Equity in the model is often set to be level for one year, and the amount of equity is determined by selecting a premium to equity ratio.

Since present values are commonly taken of flows, rather than a series of balances, calculating the PV of equity is unique to this model. The PVE must be defined carefully, since we need it to yield sensible results when equity is held over several years, or when evaluated more frequently than at the end of each year. One way to do this is first define average equity balances during each accounting period based on the equity balances at the end of each time period.

The use of the average equity balances during each quarter eliminates the problem of having too many quarter ending balances, as well as the problem with how to handle the take-down at the end of the last quarter.

One way to define PVI/PVE is to calculate an mthly effective PVI/PVE return:

\[
PVI \over PVE = \frac{\sum_{j=0}^{\infty} INC_j \ast v_{(m)}^j}{\sum_{j=0}^{\infty} EQB_j \ast v_{(m)}^j}
\]

In general, when income in each period is not a fixed multiple of the equity balances, the PVI/PVE as defined here will depend on the interest rate(s) used in discounting the premium and the equity. This leads to some disagreeable results, and the PVE undergoes dramatic changes if we shift from an annual model to a more granular level.

To address these concerns use an alternate definition in which all income is evaluated as of the end of the first year, and in which the present value of equity is put on an “annualized” basis:

\[
PVE_{ann} = \frac{\sum_{j=1}^{\infty} EQB_j \ast v_{(m)}^{j-1}}{\sum_{j=1}^{\infty} v_{(m)}^{j-1}}
\]

When equity is held fixed for one year, its annualized present value equals the “block” amount under any mthly model, irrespective of “m” or the interest rate used:

\[
PVI \over PVE_{ann} = \frac{(1+i) \sum_{j=0}^{\infty} INC_j \ast v_{(m)}^j}{PVE_{ann}}
\]

The rate used in discounting the income numerator is often selected to be the pre-tax, risk-free, new money yield. Usually, the same rate is used in the denominator, although we could also use the target rate in the denominator. While measure of risk is a problem, if the rate is risk-free, there’s no need to worry about the investment default risk. The resulting underwriting profit provision will not depend on the particular investment strategy of a company.

A different view is that we should use actual portfolio yields in conjunction with discount rates and target returns comparable to historically acceptable GAAP ROE targets.

We can use different rates for income and equity. In practice, since income is usually negative for the first few periods and then is subsequently positive, it follows that an increase in the rate for discounting income will result in a decrease of the PVI/PVE. An increase in the discount rate will always reduce the PVE. This enhances leverage, so positive PVI/PVE will be more positive, and negative PVI/PVE will be more negative. When using this model to find an underwriting profit provision consistent with a selected target return, the leverage effect will boost PVI/PVE. This leads to a reduction in the indicated underwriting profit provision, if we keep the target return fixed.

Key advantage of the method is that it’s based on a measure of return that is:

1) Comparable to GAAP ROE
2) A generalization of the standard definition of the rate of interest.

Disadvantages:

1) Requires selection of rates for calculating investment income and taking present values
2) Must choose a target return
3) May encounter debate on the choice of a target return and the explicit surplus requirement.

7. Present Value Cash Flow Return Model

This method sets the profit provision so that the present value of total cash flows equals the present value of the changes in equity. An investment rate of return is used to calculate the PV of the total cash flow, while a target rate of return is used to calculate the present value of changes in equity.

\[ PV(\Delta EQ; r) = PV(TCF; i) \]

The idea is that the target rate of return on equity can be used along with equity level assumptions to arrive at a target value for the present value of the total cash flow.

\[ TCF = UWCF + INVIEQ - FIT \]

UWCF = Underwriting Cash Flow, INVIEQ = Investment Income on Investible Equity, FIT = Federal Income Tax

Investible equity refers to the portion of the equity which can be associated with investible assets, which is approximately equal to the statutory surplus.

Income taxes can be modeled simply by applying the appropriate tax rate to the present of the underwriting cash flows and to the investment yield on investible equity. To get a more accurate treatment of taxes, we’d have to compute income according to tax accounting standards, which would increase the complexity of the model.

The primary “fault” of this model is that it’s not totally obvious what sort of profit is being measured. The Present value of total cash flows can’t be easily reconciled with GAAP accounting, since the timing of underwriting cash flows is generally not the same as the timing of GAAP underwriting income. The method has appeal, because the PV of underwriting cash flows is what most people think of when trying to measure the profit net of the associated investment income.

8. Risk-Adjusted Discounted Cash Flow Model

This method involves computing a “fair” premium, and calculating the profit provision after the fact. The fair premium is the sum of the risk adjusted present value of the underwriting cash flows, plus and amount to cover taxes (on a PV basis).

The risk adjustment is accomplished by discounting cash flows at a risk-adjusted rate that is usually less than the risk-free rate. Only the paid losses will be subject to risk-adjusted discounting. Premium, expense, and tax flows will be discounted at the risk free rate. Keep in mind we need to reflect income taxes on the surplus associated investment income.

\[ PV(P; i_r) = PV(L; i_r) + PV(FX + VX; i_f) + PV(FIT; i_f) \]

The risk adjusted rate is:

\[ i_r = i_f + \beta(i_m - i_f) \]

This quantifies the risk-return tradeoff, and is a simplified expression of the concepts underlying CAPM. The idea is that the market demands higher expected returns for riskier investments. The last term is the “market premium” for an investment of average risk. \( i_m \) is the return on an “average market portfolio.” The beta coefficient relates to the relative systematic risk of the particular investment under consideration. It’s the ratio of the covariance between the market return and the return of the particular asset, divided by the variance of the market return. (Example: If a stock moves in the same direction as the market, but with swings always twice as large, then the beta is two).

With liabilities such as loss reserves, there’s no open market to provide empirical validation for the theory. Evidence shows that the price for loss portfolio transfers is usually greater that the PV of the losses transferred (when the PV is taken at the risk free rate). This implies that the beta for liabilities is negative, but doesn’t give us any data with which to estimate the betas.

An indirect approach to estimating a liability beta is to first calculate the beta for stocks of insurance companies, and then the beta for the investment portfolios held by these companies. The difference in betas must be due to an implicit
market valuation of insurer liabilities. The problem is that most companies are multi-line companies, so we have a hard
time distinguishing betas by lines of business.

This method is intuitive, and is directly grounded in modern financial theory, so we might be able to accept that we don’t
have the “right” beta. By defining a fair premium without resorting to a total rate of return measure, we don’t need to
define the “right” target rate of return. There’s no return relative to equity, so the selection of a surplus requirement is
not as critical.

9. Internal Rate of Return on Equity Flow Model

Under this method, the profit provision is set to achieve a selected target return. Return is calculated by modeling a
single insurance policy. Using basic accounting relations, calculated flows of money between the company and its
hypothetical stockholders (equity flows). The return on the policy is the IRR on the equity flows. The IRR can be
interpreted as the interest rate paid to stockholders on a series of equity “loans” made to the insurance company.

The IRR is \( y \), so that

\[
0 = PV(x; y) = \sum_{j=0}^{n} x_j \cdot (1 + y)^{-j}
\]

The IRR is the solution to an \( n^{th} \) degree polynomial. If it exists, then the flows can be expressed as a sum of overlaid
simple loans, all carrying an interest rate equal to the IRR. We need to be careful, as the IRR only measures the interest
rate on the loans; not who is doing the lending and borrowing.

If the defining equation has more than one real root, then the IRR doesn’t exist. This isn’t usually a problem in practice.

Equity Flows:

\[
EF_j = INC_j - (SCHNG)_j
\]

The subscript connotes a flow occurring, or income accruing as of the end of the \( j \)th time period, and a balance sheet
evaluated at the end of the \( j \)th time period.

The accounting principle behind the equity flow calculation is that the surplus of the company can change only if it
declares income or if it receives or distributes funds to stockholders. Equity flows are defined so that a positive equity
flow represents a flow of money to stockholders.

What’s most important is what the stockholders will put into the company and what they will get back. Accounting rules
are important because they can affect the equity flows. The defining equity flow equation is given using Statutory
Income and Statutory Surplus, we should determine the equity flows so that the company maintains a sound balance
sheet under both accounting systems. The IRR on equity flows isn’t a GAAP return or a STAT return, it’s the return to
stockholders reflecting the constraints of both accounting systems.

In the model, the stockholders are a “bottomless well,” that gives the company funds as needed to maintain the pre-set
surplus requirement. The company immediately returns excess funds to the stockholders. It doesn’t build up profits or
retain surplus above the predetermined surplus requirement level.

The model calculates a return to stockholders that is directly analogous to the interest rate on a loan. Another strength is
that it reflects the rules of accounting, at least to the point that they impact the flow of funds to stockholders. The cost of
conservatism in STAT accounting is included.

Drawbacks are the need to select a target return and a surplus requirement. This forces us to decide how to “price” the
elements of risk in terms of increasing the target return or required surplus.

10. Conclusion

Methods don’t just vary in the details, but in the basic foundations.

Distinction 1: Whether the traditional underwriting profit is used as a starting point, or whether an attempt is made to
define the “correct” provision from first principles.

<table>
<thead>
<tr>
<th>Starting Point</th>
<th>First Principles</th>
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<tbody>
<tr>
<td>Calendar Year Investment Offset</td>
<td>All Others</td>
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<tr>
<td>Present Value Loss Offset</td>
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</tbody>
</table>
**Distinction 2:** Degree of Reliance on Calendar Year Data

<table>
<thead>
<tr>
<th>High</th>
<th>Potential Indirect*</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calendar Year Investment Offset</td>
<td>Present Value Offset</td>
<td>Risk-Adjusted Discounting</td>
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<td>Calendar Year Return</td>
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<td>Present Value Cash Flow Return</td>
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<tr>
<td>IRR on Equity Flows</td>
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* calendar year figures might be used in selecting interest rates, but market risk-free rates are preferred

**Distinction 3:** How “risk” is reflected

In the risk adjusted discounted cash flow method, pricing for risk is central. With all the total return models, risk is reflected in the selection of the target return and the surplus requirement. Risk is not considered in the models that use the traditional provision as a starting point.

**Distinction 4:** The Role of accounting convention

Constraints are central to the IRR on Equity flows method, because they impact the modeled flows of stockholder equity. GAAP ROE vs. STAT ROS is a major issue in the Calendar Year Return Procedure