For the new syllabus introduced with the July 2017 Exam.

How much detail is needed and how many problems need to be done varies by person and topic. In order to help you to concentrate your efforts:

1. About 1/6 of the many problems are labeled “highly recommended”, while another 1/6 are labeled “recommended.”
2. Important Sections are listed in bold in the table of contents.
   Extremely important Sections are listed in larger type and in bold.
3. Important ideas and formulas are in bold.
4. A section of Important Ideas and Formulas.
5. A chart of past exam questions by Section.

My Study Guide is a thick stack of paper. However, many students find they do not need to look at the textbook. For those who have trouble getting through the material, concentrate on sections in bold.
Sections and material in italics should be skipped on the first time through.

Highly Recommended problems (about 1/6 of the total) are double underlined.
Recommended problems (about 1/6 of the total) are underlined.
Do at least the Highly Recommended problems your first time through.
It is important that you do problems when learning a subject and then some more problems a few weeks later.
Be sure to do all the recent exam questions at some point.

I have written some easy and tougher problems. The former exam questions are arranged in chronological order. The more recent exam questions are on average more similar to what you will be asked on your exam, than are less recent exam questions.

In the electronic version use the bookmarks / table of contents in the Navigation Panel in order to help you find what you want. You may find it helpful to print out selected portions, such as the Table of Contents and the Important Ideas Section.

My Practice Exams are sold separately. My Seminar Slides are sold separately.

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1 The number of pages is not as important as how long it takes you to understand the material. One page in a textbook might take someone as long to understand as ten pages in my Study Guides.
2 Unfortunately, there are only a few released exams, plus the sample exam questions.
3 “Points” in my study guides are based on 100 points = a 4 hour exam. Questions on your exam are worth the equivalent of “2.5 points.”
The MFE CBT exam will provide a formula document as well as a normal distribution calculator that will be available during the test by clicking buttons on the item screen. Details are available on the Prometric Web Site. To try it out: https://www.prometric.com/en-us/clients/soa/pages/calculator.aspx

“Similar to other exam reference buttons, the normal distribution calculator button will be available throughout the exam in the top right corner of every item screen. Click the button to call up the calculator and calculate cumulative normal distribution and inverse cumulative normal distribution values. Use these values to answer the question as needed.”

“When using the normal distribution calculator, values should be entered with five decimal places. Use all five decimal places from the result in subsequent calculations.”

The normal distribution calculator button replaces the Normal Table.
The previous rule on rounding no longer applies.¹
You can try the normal distribution calculator button at the Prometric Web Site.
You will benefit from using it at least part of the time when you are studying.

The formula sheet contains the same information about the Normal and LogNormal distributions as was provided in the past.

Besides many past exam questions from the CAS and SOA, my study guides include some past questions from exams given by the Institute of Actuaries and Faculty of Actuaries in Great Britain. These questions are copyright by the Institute of Actuaries and Faculty of Actuaries, and are reproduced here solely to aid students studying for actuarial exams. These IOA questions are somewhat different in format than those on your exam, but should provide some additional perspective on the syllabus material.

I suggest you buy and try the TI-30XS Multiview calculator.
You will save time doing repeated calculations using the same formula.
The BA II Plus Professional calculator is useful for calculations involving interest.
Many people find it helpful to have both calculators during the exam.

Download from the SOA website, a copy of the tables to be attached to your exam.²
Read the “Hints on Study and Exam Techniques” in the CAS Syllabus.³
Read “Tips for Taking Exams.”⁴

¹ Unfortunately, most of my solutions were written up using the prior rule: “On Joint Exam 3F/MFE, when using the normal distribution, choose the nearest z-value to find the probability, or if the probability is given, choose the nearest z-value. No interpolation should be used. For example, if the given z-value is 0.759, and you need to find \( \Pr(Z < 0.759) \) from the normal distribution table, then chose the probability for z-value = 0.76:
\[
\Pr(Z < 0.76) = 0.7764.
\]
This should not make a significant difference.
² You will be supplied with information on the Normal and LogNormal Distributions.
³ http://casact.org/admissions/syllabus/index.cfm?fa=hints
⁴ http://www.casact.org/admissions/index.cfm?fa=tips
Starting in Spring 2011, MFE is 3 hours and given via Computer Based Testing (CBT).

While studying, you should do as many problems as possible. Going back and forth between reading and doing problems is the only way to pass this exam. The only way to learn to solve problems is to solve lots of problems. You should not feel satisfied with your study of a subject until you can solve a reasonable number of the problems.

There are two manners in which you should be doing problems. First you can do problems in order to learn the material. Take as long on each problem as you need to fully understand the concepts and the solution. Reread the relevant syllabus material. Carefully go over the solution to see if you really know what to do. Think about what would happen if one or more aspects of the question were revised. This manner of doing problems should be gradually replaced by the following manner as you get closer to the exam.

The second manner is to do a series of problems under exam conditions, with the items you will have when you take the exam. Take in advance a number of points to try based on the time available. For example, if you have an uninterrupted hour, then one might try either 60/2.5 = 24 points or 60/3 = 20 points of problems. Do problems as you would on an exam in any order, skipping some and coming back to some, until you run out of time. I suggest you leave time to double check your work.

Expose yourself somewhat to everything on the syllabus. Concentrate on sections and items in bold. Do not read sections or material in italics your first time through the material. My chart of where the past exam questions have been may also help you to direct your efforts.

Try not to get bogged down on a single topic. On hard subjects, try to learn at least the simplest important idea. The first time through do enough problems in each section, but leave some problems in each section to do closer to the exam. Make a schedule and stick to it. Spend a minimum of one hour every day. I recommend at least two study sessions every day, each of at least 1/2 hour.

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8 Some may also find it useful to read about a dozen questions on an important subject, thinking about how to set up the solution to each one, but only working out in detail any questions they do not quickly see how to solve.

9 Material in italics is provided for those who want to know more about a particular subject and/or to be prepared for more challenging exam questions; it could be directly needed to answer perhaps one question on an exam.

10 While this may indicate what ideas questions on your exam are likely to cover, every exam contains a few questions on ideas that have yet to be asked. Your exam will have its own mix of questions.
Throughout do Exam Problems and Practice Problems in my study guides. At least 50% of your time should be spent doing problems. As you get closer to the Exam, the portion of time spent doing problems should increase. Review the important formulas and ideas sections, at the end of each study guide. During the last several weeks do my practice exams, sold separately.

Here is a schedule that may help some people go through my study guide.¹¹

**A 12 week Study Schedule for Exam MFE:**

1. Sections 1-4
2. Sections 5-8

3. Sections 9-15
4. Sections 16-22

5. Sections 23-27
6. Sections 28-31

7. Sections 32-35
8. Sections 36-44

9. Sections 45-49
10. Sections 50-56

11. Sections 57-61
12. Sections 62-66

Throughout, go back and review the important ideas and do some more problems in sections you have gone through previously.

¹¹ This is just an example of one possible schedule. Adjust it to suit your needs or make one up yourself.
Past students helpful suggestions and questions have greatly improved this study guide.

I thank them! Feel free to send me any questions or suggestions:

**Howard Mahler, Email: hmahler@mac.com**

Please do not copy the Study Guide, except for your own personal use. Giving it to others is unfair to yourself, your fellow students who have paid for them, and myself. If you found them useful, tell a friend to buy his own.

Please send me any suspected errors by Email prior to the exam.
(Please specify as carefully as possible the page, Study Guide, and Exam.)

**Author Biography:**

Howard C. Mahler is a Fellow of the Casualty Actuarial Society, and a Member of the American Academy of Actuaries. He has taught actuarial exam seminars and published study guides since 1994.

He spent over 20 years in the insurance industry, the last 15 as Vice President and Actuary at the Workers' Compensation Rating and Inspection Bureau of Massachusetts. He has published many major research papers and won the 1987 CAS Dorweiler prize. He served 12 years on the CAS Examination Committee including three years as head of the whole committee (1990-1993).

Mr. Mahler has taught live seminars and/or classes for Exam C, Exam MFE, CAS Exam S, CAS Exam 5, and CAS Exam 8. He has written study guides for all of the above.

hmahler@mac.com  www.howardmahler.com/Teaching

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12 This study guide represents thousands of hours of work.
## Exam MFE Study Guide

### Pass Marks and Passing Percentages for Past Exams

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13 Information taken from the SOA webpage. Check the webpage for updated information.

14 “Starting in May 2011, Exam 3F/MFE is administered using computer-based testing (CBT). Under CBT, it is not possible to schedule everyone to take the examination at the same time. As a result, each administration consists of multiple versions of the examination given over a period of several days. The examinations are constructed and scored using Item Response Theory (IRT). Under IRT, each operational item that appears on an examination has been calibrated for difficulty and other test statistics and the pass mark for each examination is determined before the examination is given. All versions of the examination are constructed to be of comparable difficulty to one another. For the May 2011 administration of Examination MFE/3F, an average of 71% correct was needed to pass the exam.”
Mahler’s Guide to
Financial Economics

Exam MFE

For the new syllabus introduced with the July 2017 Exam

prepared by
Howard C. Mahler, FCAS
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Study Aid 2017-MFE

Howard Mahler
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Mahler’s Guide to Financial Economics

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Concepts in Derivatives Markets by Robert L. McDonald are demonstrated.

Information in bold or sections whose title is in bold are more important for passing the exam. Larger bold type indicates it is extremely important. Information presented in italics (and sections whose titles are in italics) should not be needed to directly answer exam questions and should be skipped on first reading. It is provided to aid the reader’s overall understanding of the subject, and to be useful in practical applications.

Highly Recommended problems are double underlined.
Recommended problems are underlined.
Solutions to the problems in each section are at the end of that section.

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1 All references are to the third edition.
2 Note that problems include both some written by me and some from past exams. The latter are copyright by the Society of Actuaries and the Casualty Actuarial Society and are reproduced here solely to aid students in studying for exams. The solutions and comments are solely the responsibility of the author; the SOA and CAS bear no responsibility for their accuracy. While some of the comments may seem critical of certain questions, this is intended solely to aid you in studying and in no way is intended as a criticism of the many volunteers who work extremely long and hard to produce quality exams.
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My practice exams and my seminar slides are each sold separately.
Throughout I make many references to Derivatives Markets by McDonald; these are to the third edition. One does not need the textbook in order to use my study guide; the references are to help those who are also using the textbook.

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Unless otherwise stated chapter appendices are not included in the required readings from this text.

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\(^3\) Sections 5.1-5.2, Section 5.3 (through the middle of p.136), Section 5.4 (through the top of p.143).

\(^4\) Excluding “Options on Commodities” on pages 315 and 316.

\(^5\) Including Appendix 11.A.

\(^6\) Including Appendices 12.A and Appendix 12.B.

\(^7\) Sections 14.1-14.3, Section 14.4 (through the bottom of p.419), Sections 14.5-14.6.

\(^8\) Including Appendix 18.A.

\(^9\) But with only those definitions in Tables 23.1 and 23.2 that are relevant to Section 23.1, including the top half of p.714 (Re: Lookback calls and puts).

\(^10\) Section 25.1 (through the bottom of p.754), Section 25.4 (through the middle of p.773), Section 25.5 (through the middle of p.781).
This study guide covers all of the material on SOA Exam MFE. The syllabus consists of various sections of the 3rd edition of Derivatives Markets by Robert L. McDonald.

The SOA has provided sample questions in two separate files. The first file contains questions on “Introductory Derivatives”, covers material formerly on Exam MFE. The second file contains questions on “Advanced Derivatives”.

Unless stated otherwise in a question assume:

- The market is frictionless. There are no taxes, transaction costs, bid/ask spreads, or restrictions on short sales. All securities are perfectly divisible. Trading does not affect prices. Information is available to all investors simultaneously. Every investor acts rationally (i.e., there is no arbitrage.)
- The risk-free rate is constant
- The notation is the same as used in Derivatives Markets by Robert L. McDonald.
- The MFE CBT exam will provide a formula document as well as a normal distribution calculator that will be available during the test by clicking buttons on the item screen. Details are available on the Prometric Web Site. http://www.prometric.com/SOA/MFE3F_calculator.htm

“Similar to other exam reference buttons, the normal distribution calculator button will be available throughout the exam in the top right corner of every item screen. Click the button to call up the calculator and calculate cumulative normal distribution and inverse cumulative normal distribution values. Use these values to answer the question as needed.”

“When using the normal distribution calculator, values should be entered with five decimal places. Use all five decimal places from the result in subsequent calculations.”

The normal distribution calculator button replaces the Normal Table. The previous rule on rounding no longer applies.

You can try the normal distribution calculator button at the Prometric Web Site. You will benefit from using it at least part of the time when you are studying.

11 The portions of Chapters 1,2,3, and 5 of Derivative Markets that are on the syllabus. While the question numbers go to 75, the SOA has deleted some questions. Deleted questions: 4, 18, 19, 22, 23, 27, 28, 31, 34, 36, 54, 57, 58, 63, 64.

12 These were formerly MFE Sample questions. While the question numbers go to 76, the SOA has deleted some questions. Deleted questions: 10-14, 16, 21-24, 27, 32, 34-38, 43, 45, 48, 56, 60-74.

13 Unfortunately, most of my solutions were written up using the prior rule: “On Joint Exam 3F/MFE, when using the normal distribution, choose the nearest z-value to find the probability, or if the probability is given, choose the nearest z-value. No interpolation should be used. For example, if the given z-value is 0.759, and you need to find \( \Pr(Z < 0.759) \) from the normal distribution table, then chose the probability for z-value = 0.76: \( \Pr(Z < 0.76) = 0.7764. \)” This should not make a significant difference.
The formula sheet contains the same information about the Normal and LogNormal distributions as was provided in the past, as reproduced below.

Unless otherwise stated in the examination question, assume:
- The market is frictionless. There are no taxes, transaction costs, bid/ask spreads or restrictions on short sales. All securities are perfectly divisible. Trading does not affect prices. Information is available to all investors simultaneously. Every investor acts rationally and there are no arbitrage opportunities.
- The risk-free interest rate is constant.
- The notation is the same as used in Derivatives Markets, by Robert L. McDonald.

When using the normal distribution calculator, values should be entered with five decimal places. Use all five decimal places from the result in subsequent calculations.

In Derivatives Markets, $\Pr(Z < x)$ is written as $N(x)$.

The standard normal density function is
\[
f_Z(x) = N'(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} = \frac{e^{-x^2/2}}{\sqrt{2 \times 3.14159}} = \frac{e^{-x^2/2}}{2.50663}, \quad -\infty < x < \infty.
\]

Let $Y$ be a lognormal random variable. Assume that $\ln(Y)$ has mean $m$ and standard deviation $\nu$. Then, the density function of $Y$ is
\[
f_Y(x) = \frac{1}{x\nu\sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\ln(x) - m}{\nu}\right)^2\right], \quad x > 0.
\]

The distribution function of $Y$ is
\[
F_Y(x) = N\left(\frac{\ln(x) - m}{\nu}\right), \quad x > 0.
\]

Also,
\[
E[Y^k] = \exp\left(km + \frac{1}{2} k^2 \nu^2\right),
\]
which is the same as the moment-generating function of the random variable $\ln(Y)$ evaluated at the value $k$. 
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The SOA did not release its 11/07 exam MFE.
The CAS/SOA did not release the 5/08, 11/08, 11/09, and subsequent exams 3F/MFE.

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Section 1, Introduction

There are a number of basic ideas that will be used and discussed in subsequent sections.

**Continuously Compounded Risk Free Rate:**

If \( r \) is the continuously compounded annual risk free rate, then the present value of \$1 T\ years in the future is: \( e^{-rT} \).

A continuously compounded rate is what an actuary would call the force of interest.

If the continuously compounded annual risk free rate is 5%, then the present value of \$100 two years from now is: \$100 \cdot e^{-(2)(5\%)} = \$90.48.

**Effective Annual Rate:**

If \( r \) is the effective annual risk free rate, then the present value of \$1 T\ years in the future is: \( 1/(1+r)^T \). An effective annual rate is what an actuary would call the rate of interest.

If the effective annual risk free rate is 5%, then the present value of \$100 two years from now is: \$100 / 1.05^2 = \$90.70.

Effective annual rates will be used in Interest Rate Caps and the Black-Derman-Toy Model, to be discussed in subsequent sections. Otherwise, mostly we will use continuously compounded rates.

**Derivatives:**

A derivative is an agreement between two people that has a value determined by the price of something else. Examples are options, forward contracts, and futures contracts.

For example, Alan gives Bob the right to buy from Alan a share of IBM stock one year from now at a price of \$120. This is an example of a stock option. The value of this option depends on the price of IBM stock one year from now.

---

14 See Appendix B.1 of *Derivatives Markets* by McDonald.
15 See Appendix B.1 of *Derivatives Markets* by McDonald.
16 Warren E. Buffett has said, “Derivatives are financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal.”
**Options:**

A call is an option to buy. For example, Bob purchased a call option on IBM stock from Alan.

A put is an option to sell. For example, if Debra purchased a put option on IBM stock from Carol, then Debra will have the option in the future to sell a share of IBM stock to Carol at a specified price.

**Forward Contracts:**

As will be discussed in a subsequent section, a forward contract is an agreement that sets the terms today, but the buying or selling of the asset takes place in the future. For example, Clark agrees to sell Lois 100 shares of ABC stock for $60 per share one year from now.

The purchaser of an option has bought the right to do something in the future, but has no obligation to do anything. In contrast, in a forward contract both parties are obligated to fulfill their parts of the contract.

**Futures Contracts:**

As will be discussed in a subsequent section, futures contract is similar to a forward contract except: A futures contract is typically traded on an exchange.

A futures contract is marked to market periodically. The buyer and the seller post margin.

**Continuous Dividends:**

We often assume that dividends are paid at a continuous rate $\delta$. Over a short period of time $dt$, stock dividends of: $\delta S(t) \, dt$ are paid, where $S(t)$ is the stock price at time $t$.

So if one buys a share of stock at time 0, and reinvests the dividends in the stock, at time $T$ one would have $e^{T \delta}$ shares of the stock.

Exercise: One buys 1 million shares of a stock that pays dividends at the continuous annual rate of 2%. The dividends are reinvested in that stock.

After 3 years how many shares of the stock does one own?

[Solution: $(1 \text{ million}) e^{(3)(0.02)} = 1,061,837$ shares.]

---

17 Marked-to-market means the item is revalued to reflect current market prices.
18 A deposit which compensates the other party to a futures contract in case one of the parties does not fulfill its obligation.
19 This is a good approximation for a stock index fund.
20 $\delta$ acts similarly to a force of interest.
Continuously Compounded Returns:

Let \( S_t \) and \( S_{t+h} \) be the stock prices at times \( t \) and \( t+h \). Then the continuously compounded return on the stock between time \( t \) and \( t+h \) is: \( \ln\left[ \frac{S_{t+h}}{S_t} \right] \). On an annual basis, this return is: \( \frac{\ln\left[ \frac{S_{t+h}}{S_t} \right]}{h} \).

For example, if the stock price is $80 at time 0 and $90 at time 2 years, then the continuously compounded return from time 0 to 2 is: \( \ln\left[ \frac{90}{80} \right] = 11.78\% \). On an annual basis, this return is: \( \frac{11.78\%}{2} = 5.89\% \).

Exercise: The stock price is $90 at time 2 years and $85 at time 2.5 years, what is the annual continuously compounded return?
[Solution: \( \ln\left[ \frac{85}{90} \right] / 0.5 = -11.4\% \).]

One can get the future stock price from the current stock price and the continuously compounded return. For example, if the current stock price is $100, and the continuously compounded return over the next three years is 8% per year, then the stock three years from now is: \( 100 \cdot e^{0.24} = $127.12 \).

Exercise: The current price of a stock price is $60. Over the next four years the annual continuously compounded return are: 17%, 33%, -140%, and 6%. What is the stock price in four years?
[Solution: \( 60 \cdot e^{0.17} \cdot e^{0.33} \cdot e^{-1.40} \cdot e^{0.06} = 60 \cdot e^{-0.84} = $25.90 \).
Comment: The continuously compounded returns add; the return over the whole four years is: \( 17\% + 33\% - 140\% + 6\% = -84\% \). When the stock price declines by a very large amount, one can have a continuously compounded return of less than -100%.]

Volatility:

The volatility of a stock is the standard deviation of its continuously compounded returns.\(^{21}\)

Actuarial Present Values:

Let us assume that one year from now an insurer will pay either $50 with probability 70% or $100 with probability 30%. Then the expected payment in one year is: \( (0.7)(50) + (0.3)(100) = $65 \).

Assume that the continuously compound annual rate of interest is now 5%.
Then the actuarial present value of the insurer’s payment is: \( 65 \cdot e^{-0.05} = $61.83 \).\(^{22}\)

\(^{21}\) Volatility will be discussed in subsequent sections and is usually stated on an annual basis.
\(^{22}\) If instead the 5% were an effective annual rate, then the actuarial present value would be: \( 65/1.05 = $61.90 \).
In general in order to calculate an actuarial present value, one takes a sum of the expected payments at each point in time each multiplied by the appropriate discount factor. The discount factor adjusts for the difference between the time value of money at the present and at the time when the payment is made.

Exercise: In addition to the payments one year from now, the insurer will pay two years from now either $50 with probability 50%, $100 with probability 40%, or $200 with probability 10%. Assume that one year from now the continuously compound annual rate of interest will be 6%. Determine the actuarial present value of the insurer’s total payments, including those made one year from now and two years from now.

[Solution: The expected payment in two years is: $(0.5)(50) + (0.4)(100) + (0.1)(200) = 85$. Discounting back to the present: $85 \exp[-0.05 - 0.06] = 76.15$. Adding in the actuarial present value of the payments made in one year, the actuarial present value of the insurer’s total payments is: $61.83 + 76.15 = 137.98$.]

**Long and Short Positions:**

Entering into a long position is buying. Entering into a short position is selling or writing.

More generally, a position is long with respect to an underlying asset (stock or commodity) if it becomes more valuable when the price of the underlying increases. For example, buying a stock is long with respect to the stock. As will be discussed if one buys a forward contract on a stock, then your position increases in value if the stock price increases; thus, a purchased forward contract is long with respect to the underlying asset. Other long positions include buying a call option on a stock.

In contrast, a position is short with respect to an underlying asset if it becomes less valuable when the price of the underlying increases. For example, shorting a stock or selling a forward contract on a stock are each short with respect to the stock. Other short positions include buying a put option on a stock.
Selling Short:

If we sell a stock short, then we borrow a share of stock and sell it for the current market price. We will give this person a share of stock at the designated time in the future. We also must pay this person any stock dividends they would have gotten on the stock, when they would have gotten them.

Ozzie borrows 1000 shares of stock from Harriet and sells them. The stock pays dividends at a continuously compounded annual rate of 1%. If Ozzie agreed to return the shares in six months, then he would return to Harriet $1000 e^{1\%/2} = 1005$ shares.

Arbitrage:

If there is a possible combination of buying and selling with no net investment that has no risk but generates positive (or at least nonnegative) cashflows, this is an arbitrage opportunity. Taking advantage of such an opportunity is called arbitrage. In other words, arbitrage is free money.

If an arbitrage opportunity existed, clever traders would take advantage of it. Relatively quickly, the prices would adjust so as to remove this opportunity for arbitrage.

Generally, we assume prices should be such that they do not permit arbitrage. In other words, we assume that there is no free lunch. This is called “no-arbitrage” pricing.

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2017-MFE, Financial Economics §1 Introduction, HCM 3/15/17, Page 13

23 Buying the shares and returning them is called closing or covering the short position.
Bonds:

A bond pays its owner a fixed amount when it matures, plus fixed payments at fixed intervals, termed coupons. For example a two-year 12% coupon bond might pay:

<table>
<thead>
<tr>
<th>Time</th>
<th>Payment to Owner</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>1.5</td>
<td>6</td>
</tr>
<tr>
<td>2.0</td>
<td>106</td>
</tr>
</tbody>
</table>

In contrast, a zero-coupon bond only pays a fixed amount at maturity, called the par value of the bond. For example, a two-year zero-coupon bond with a par value of $100 would pay its owner $100 in two years. We will only consider zero-coupon bonds on this exam.

Exercise: A zero-coupon bond matures in two years and pays $100. The current price of this bond is $93.

What is the effective annual interest rate?
What is the continuously compounded annual interest rate?

[Solution: \(93 (1+r)^2 = 100. \Rightarrow r = 3.70\% \text{ effective.}\)

\[93 e^{2r} = 100. \Rightarrow r = 3.63\% \text{ continuously compounded.}\]

Bonds, Borrowing, and Lending:

Barry buys a bond from IBM.
Barry gives money to IBM in exchange for receiving future payments.
Barry is lending money.

IBM issues (sells) a bond to Barry.
IBM receives money from Barry in exchange for making future payments.
IBM is borrowing money.

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\(^{24}106 = \text{principal of 100 plus a coupon of 6.}\)
Problems:

1.1 (2 points) A zero-coupon bond matures in three years and pays $1000. The current price of this bond is $875.
(a) What is the effective annual interest rate?
(b) What is the continuously compounded annual interest rate?

1.2 (1 point) One buys 3000 shares of a stock that pays dividends at the continuous annual rate of 1%. The dividends are reinvested in that stock. After 2 years how many shares of the stock does one own?

1.3 (1 point) The continuously compounded annual risk free rate is 6%. What is the present value of $5000 to be received 8 months from now?

1.4 (1 point) The current price of a stock price is $80. Over the next four months, the annual continuously compounded return are: 2%, -3%, 4%, and 8%. What is the stock price in four months?

1.5 (1 point) The effective annual risk free rate is 7%. What is the present value of $200 to be received 4 months from now?

1.6 (1 point) Two years from now, an insurer will pay either: $1000 with probability 60%, $2000 with probability 30%, or $5000 with probability 10%. The continuously compound annual rate of interest will be 8%. Determine the actuarial present value of the insurer’s payment.

1.7 (2 points) A zero-coupon bond matures in six months and pays $100. The current price of this bond is $98.
(a) What is the effective annual interest rate?
(b) What is the continuously compounded annual interest rate?

1.8 (2 points) Define and briefly discuss short selling stock.

1.9 (1 point) Define a derivative and give two examples.
1.10 (CAS3, 11/07, Q.25) (2.5 points) On January 1, 2007, the Florida Property Company purchases a one-year property insurance policy with a deductible of $50,000. In the event of a hurricane, the insurance company will pay the Florida Property Company for losses in excess of the deductible. Payment occurs on December 31, 2007. For the last three months of 2007, there is a 20% chance that a single hurricane occurs and an 80% chance that no hurricane occurs. If a hurricane occurs, then the Florida Property Company will experience $1,000,000 in losses. The continuously compounded risk-free rate is 5%. On October 1, 2007, what is the risk-neutral expected value of the insurance policy to the Florida Property Company?
A. Less than $185,000
B. At least $185,000, but less than $190,000
C. At least $190,000, but less than $195,000
D. At least $195,000, but less than $200,000
E. At least $200,000
Solutions to Problems:

1.1. (a): 875 \((1+r)^3\) = 1000. \(\Rightarrow r = 4.55\%\) effective.
(b) 875 \(e^{3r}\) = 1000. \(\Rightarrow r = 4.45\%\) continuously compounded.

1.2. 3000 \(e^{(0)(0.01)}\) = 3061 shares.

1.3. $5000 \(e^{-(2/3)(6\%)}\) = $4804.

1.4. 80 \(\exp[0.02/12]\) \(\exp[-0.03/12]\) \(\exp[0.04/12]\) \(\exp[0.08/12]\) = $80.74.

1.5. $200 / 1.07^{4/12} = $195.54.

1.6. The expected payment in two years is: \((0.6)(1000) + (0.3)(2000) + (0.1)(5000)\) = $1700. Discounting back to the present: \(1700 \exp[-(2)(8\%)]= $1449\).

1.7. (a) 98 \((1+r)^{0.5}\) = 100 \(\Rightarrow r = 4.12\%\) effective.
(b) 98 \(e^{r/2}\) = 100. \(\Rightarrow r = 4.04\%\) continuously compounded.

1.8. If we sell a stock short, then we borrow a share of stock and sell it for the current market price. We will give this person a share of stock at the designated time in the future. We also must pay this person any stock dividends they would have gotten on the stock, when they would have gotten them.

1.9. A derivative is an agreement between two people that has a value determined by the price of something else. Examples include: options (puts and calls), forward contracts, and futures contracts.

1.10. B. We subtract the deductible of $50,000; 1,000,000 - 50,000 = 950,000.
\((20\%)(950,000) / e^{0.05/4} = $187,640\).
Comment: This question has nothing to do with derivatives. You are merely being asked to take an actuarial present value using a continuously compound risk-free rate (a force of interest.)
Section 2, Financial Markets and Assets

McDonald provides some useful background on financial markets and financial assets. However, since this is an exam for actuaries, I think you are less likely to be tested on the details.

Trading Financial Assets:

- The buyer and seller must agree on a price.
- The trade must be cleared; the obligations of each party are specified.
- The trade must be settled.
- Ownership records must be updated.

Trades usually take place on organized exchanges. Stock exchanges trade stocks. Bond markets trade bonds. One can trade directly with a dealer in the over-the-counter (OTC) market, rather than on an exchange.

Derivatives exchange trade derivatives. Derivative markets were started in the 1970s, which coincided with an increase in variability of prices. This increase in price risk can be lessened by the use of derivatives. The types of derivatives and the volumes traded have increased since their introduction.

After a trade has taken place, a clearinghouse matches the buyers and sellers. A derivative clearinghouse will typically interpose itself in the transaction, becoming the buyer to all sellers and the seller to all buyers; this is called novation.

Measures of Market Size and Activity:

- Trading volume: the number of shares or options traded on an exchange.
- Market Value: the value of all of the outstanding stock of a firm.
- Notational Value: the value of the assets underlying a position.
- Open Interest: the number of contracts on a derivatives exchange.

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25 See Sections 1.2, 1.3, and 1.5 of Derivative Markets by McDonald
26 For example, currencies were allowed to float with respect to each other, so that exchange rates vary. The price of oil has become more variable. The 3-month Treasury bill rate has become more variable month to month. See Figure 1.1 in Derivative Markets by McDonald.
27 A recent development is the growth of prediction markets.
28 For example, the market value of Apple Computer might be $600 billion.
29 For example, if one has a call option on 100 shares of stock with a current price of $60 per share, then the notational value is $60,000. A futures contract for 100 ounces of gold, when gold futures are trading at $1100, would have a notional value of $110,000.
30 For example, on a derivatives exchange there may currently be 130,000 open interest in futures contracts for August delivery of gold. (Each contract involves 100 ounces of gold.) Each contract has two parties.
Examples of Impacts of Financial Markets:

- Mutual funds.
- Homeowners Insurance.
- Home Mortgages.
- Employers raising money in global markets.
- Firms using derivatives to protect against adverse changes in currency, interest rates, commodity prices, etc.

Risk Sharing:

Insurance is a way to share risk. For example, those who are unlucky enough to have a automobile accident during a year have their losses paid; the money to pay for these losses comes from the premiums paid by everyone, including those who did not have an auto accident this year.

If the Euro gains value compared to the dollar, then some firms will be helped while others will be hurt. The use of derivatives on currency would lessen the risk of all these firms from a change in the exchange rate. Firms who would be hurt by an increase in the value of the Euro, could enter a futures contract to buy Euros at a fixed dollar amount. Firms who would be hurt by an decrease in the value of the Euro, could enter a futures contract to sell Euros at a fixed dollar amount. Then these two types of firms are sharing the risk of a change in this exchange rate.

Reinsurance:

Insurance companies face risk themselves. For example, if a large hurricane hits Florida, then an insurer might have to pay billions of dollars on homeowners insurance claims. To protect against such risks, an insurer can buy protection from a reinsurer.\(^{31}\) Reinsurers pool such insurance risks, and are a vital part of maintaining healthy insurance markets.

\(^{31}\) This would be called catastrophe reinsurance. There are many other types of reinsurance. See for example, “Basics of Reinsurance Pricing,” by David R. Clark, CAS Study Note, on the CAS advanced ratemaking exam.
Catastrophe Bonds:

An insurer or reinsurer could issue catastrophe bonds, related to a specific type of catastrophe risk.\textsuperscript{32} For example, an insurer might issue catastrophe bonds, related to a hurricane hitting Florida and causing at least $1 billion of damage to the insurance industry.\textsuperscript{33} The buyers of the bond would receive interest payments above the market rate.\textsuperscript{34} However, if a hurricane hits Florida and causes at least $1 billion of damage to the insurance industry, then the bonds stop paying interest and the buyers of the bonds would lose their principal.\textsuperscript{35}

The insurer has transferred the risk of catastrophe to the buyers of the cat bonds. If the hurricane does not hit they earn more than the market rate on their investment. If instead a hurricane hits, then the buyers of the bonds lose their investment; the insurer can use the money paid to them for the bonds to pay insurance claims caused by the catastrophe.

Diversifiable versus Undiversifiable Risk:

Investors in stocks face risk. Diversifiable risk is specific to a company, such as a labor strike, a natural disaster, or slumping sales. By buying many different stocks, an investor can greatly reduce diversifiable risk.

On the other hand, some events can affect all firms at the same time. Events such as inflation, war, and fluctuating interest rates influence the entire economy, not just a specific firm or industry. Diversification cannot eliminate the risk of facing these events. Therefore, these are considered undiversifiable risk. This type of risk accounts for most of the risk in a well-diversified investment portfolio. The risk of lightening hitting a particular factory is diversifiable. The risk of a stock market crash is undiversifiable.

Exercise: A company’s headquarters building that it owns is located near a major earthquake fault. Is this a diversifiable risk?
[Solution: Yes. For example, the company can buy earthquake insurance.]

Exercise: An asteroid 10 miles or more across might hit the earth in the future. Is this a diversifiable risk?
[Solution: No! Such an event is likely to destroy human civilization.]

Financial markets allow diversifiable risks to be shared, and undiversifiable risk to held by those most willing to do so. The existence of these risk sharing mechanisms benefits everyone.

\textsuperscript{32} There are many details, which are beyond the syllabus of this exam.
\textsuperscript{33} The contract would contain detailed language on exactly what is a triggering event.
\textsuperscript{34} Actuaries are involved in the difficult problem of determining an appropriate rate for the bonds to earn.
\textsuperscript{35} Things may be more complicated for a particular catastrophe bond.
Details of Buying and Selling Stock:

At any given time a stock will have two prices. The price at which you can buy is the ask (offer) price. The lower price at which you can sell is the bid price. The bid-ask spread is the difference, which allows the market maker in a stock to earn a living.

For example, the ask price for Sincere Trust Company stock might be $80.25 per share while the bid price is $79.75 per share. The market maker (broker) sells to individuals at the higher $80.25 and buys from individuals at the lower $79.25. The market maker earns money from the $0.50 bid-ask spread.

Note that the names of the prices are from the point of view of the market maker. The ask or offer price is what the market maker is asking from someone for stock, while the bid price is what the market maker is willing to pay someone for stock.

Millie buys 100 shares of Sincere Trust Company. She would pay the $80.25 per share bid price, for a total of $8025. One would be charged a commission. For example, if the commission were 0.4%, Millie would pay an additional: (0.4%)(8025) = $32.10.

Assume that for some reason, Millie immediately sold her 100 shares. She would receive the bid price of $79.75 per share for total of $7975. She would again pay a commission, in this case: (0.4%)(7975) = $31.90.

Millie will have paid a total of: 8025 + 32.10 + 31.90 = $8089. Thus due the bid-ask spread and the commissions, her roundtrip transaction costs are: $8089 - 7975 = $114.

Exercise: The bid price of a stock is $120 per share, while the ask price is $121 per share. Cramer buys 1000 shares of the stock, and pays commissions of 0.5%. Cramer turns around and immediately sells his shares, and pays commissions of 0.5%. What is his round trip transaction costs.

[Solution: He pays (1000)(121) = $121,000 for his shares. He pays commissions of: (0.5%)(121,000) = $605. He now sells his shares for (1000)(120) = $120,000. He pays commissions of: (0.5%)(120,000) = $600. His roundtrip transaction costs are: 121,000 + 605 + 600 - 120,000 = $2205.]

In most exam questions, we ignore these types of real world details.\(^{36}\)

\(^{36}\) Unless stated otherwise in a question assume: The market is frictionless. There are no taxes, transaction costs, bid/ask spreads, or restrictions on short sales. All securities are perfectly divisible. Trading does not affect prices. Information is available to all investors simultaneously. Every investor acts rationally (i.e., there is no arbitrage.)
Ways to Buy or Sell Stock:

Different types of orders one can give to a stock broker when one wants to buy or sell stock:
- Market Order: trade at the best price available.
- Buy Limit Order: buy only if the price is at most a certain specified value.
- Sell Limit Order: sell only if the price is at least a certain specified value.
- Stop Loss Order: sell if and when the price drops to a certain level.

Dividends:

Some stocks pay dividends.\textsuperscript{37} For example a stock may pay a dividend of $2 to whoever owns the stock on September 15. If Mulder owns the stock on September 15, then Mulder will receive the dividend of $2. The dividend may actually be paid September 20.

If Scully buys the stock from Mulder on September 16, Scully will not receive this dividend. Scully will pay the ex-dividend price.\textsuperscript{38} If instead Scully had bought the stock on September 10, then Scully would get this dividend. In that case Scully would have paid the cum-dividend price.

All other things being equal, we expect the ex-dividend price to be less than the cum-dividend price for the stock, with the expected difference being the size of the dividend being paid.

Examples and exam questions may include dividends paid at discrete points in time, such as once every three months, as occurs in the real world. As discussed previously, they may instead include a continuous approximation, where dividends are paid at a continuously compounded rate that depends on the stock price. For example, we might assume that dividends are paid at a continuously compounded annual rate of 0.8%.

\textsuperscript{37} Dividends are declared by the board of directors of the company whose stock it is.

\textsuperscript{38} Ex-dividend is a classification of trading shares when a declared dividend belongs to the seller rather than the buyer. A stock will be given ex-dividend status if a person has been confirmed by the company to receive the dividend payment. When a company decides to declare a dividend, its board of directors establishes a record date. This is the date when a person must be on the company’s record as a shareholder to receive the dividend payment. Once the record date is set, the ex-dividend date is set according to the rules of the stock exchange on which the stock is traded. The ex-dividend date is typically set for two business days prior to the record date. Investors need to buy the dividend-paying stock at least three days before the record date, since trades take three days to settle. Since the ex-dividend date is usually set two business days prior to the record date, investors need to own the stock one day before the ex-dividend date to receive the dividend.

If a company issues a dividend in stock instead of cash, the ex-dividend date rules are slightly different. With a stock dividend, the ex-dividend date is set on the first business day after the stock dividend is paid out.
Short Selling:

As discussed previously, if we sell a stock short, then we borrow shares of stock and sell them for the current market price. We will return the shares of stock at the designated time in the future. We also must pay this person any stock dividends they would have gotten on the stock, when they would have gotten them. One can also short sell bonds, or commodities such as wine or silver.

The cashflows from short selling are the negative of those from purchasing a share of stock. When you buy a stock you pay the price per share; when you short sell, you get the price per share. If you sell stock, you get the price per share at that future time; when you conclude your short sale, you need to buy the stock at the price per share that applies at that time.

Let us assume Ike borrows 100 shares of ABC stock from Tina. The current price of ABC stock is $50. Ike sells the stock for $5000.\(^3\) Ike agrees to return buy 100 shares and give them to Tina in one year.\(^4\)

Exercise: Ignoring transaction costs and the time value of money, what is Ike’s payoff in one year? [Solution: \((100) (50 - S_1)\), where \(S_1\) is the price per share in one year.

For example, if in one year the price per share is $40, then Ike makes: \((100)(50 - 40) = 1000.\) If instead in one year the price per share is $60, then Ike makes: \((100)(50 - 60) = -1000.\)

While the owner of a stock wants the stock price to increase, a short seller instead wants the stock price to decrease.

There are three reasons to short sell: speculation, financing, and hedging. The short seller can speculate that the stock price will go down and he will make money. The short sale is a way to borrow money.\(^5\) A short sale can offset the risk of owning a stock or derivative on that stock.\(^6\)

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\(^3\) Ignoring any transaction cost paid by Ike.

\(^4\) If the stock pays dividends, then Ike agrees to pass those dividend payments on to Tina when they occur.

\(^5\) This is common in the bond market.

\(^6\) This is often done by market makers.
Details of Short Selling:

In subsequent sections, we will usually ignore the details of short selling. However, McDonald does discuss these details in his Introduction, which is on the syllabus.

Tina would face credit risk that Ike would be unwilling or able to keep his part of the bargain in one year. Therefore, the proceeds would either be given to Tina to hold or more likely be put in an escrow account. What if the price of the stock in 1 year is $60? Then the $5000 held in escrow would not be enough for Ike to buy 100 shares of stock.

Thus the short seller is usually required to deposit an additional amount beyond the proceeds of the short sale. For example, in this case, Ike might have to deposit an additional $1000 in the escrow account. This extra amount is called a haircut. The need to come up with and deposit this extra collateral limits the amount of short selling any one individual can do.

In one year, when Ike returns the stock to Tina, Ike is going to want his money being held as collateral back plus interest. The rate of interest Ike is credited on his money, the proceeds of the sale plus haircut, is called the short rebate.

For example, if the market rate of interest were 6%, then Ike may only be credited with 4% interest. This would represent a cost to Ike for short selling, in addition to any transaction costs of buying and selling stock. The size of the short rebate is a matter that would be negotiated between the parties at the start. If the supply of people like Tina willing to lend ABC stock exceeds the demand of people like Ike who want to borrow ABC stock, then Ike can probably negotiate a higher short rebate, than if the reverse is true.

Exercise: Ike pays commissions of 0.4% when he buys and sells stock. He is required to deposit the proceeds of his short sale of ABC stock in an escrow account plus a haircut of $1000. Ike’s escrow account is credited with continuously compounded interest at a rate of 4%; the risk free rate is 6%. If the stock price in one year is $45, what is Ike’s profit?

[Solution: Ike sells the 100 shares for $5000 and deposits it in the escrow account. He also deposits the $1000 haircut. He pays a commission of: (0.4%)(5000) = $20. In one year he buys 100 shares of stock for $4500, and returns the stock to Tina. He pays a commission of: (0.4%)(4500) = $18. Ike gets the money in the escrow account plus interest: 6000 e^{0.04} = $6244.86. Ike profit in one year is: 6244.86 - 1020 e^{0.06} - 4500 - 18 = $643.79.

Comment: For simplicity, we have ignored the bid-ask spread.]

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43 Escrow refers to money held by a third-party such as a bank on behalf of transacting parties.
44 This would provide Tina some protection, but would not totally protect her against the risk of the stock price increasing a lot.
45 When instead short selling bonds, this would be called the repo rate.
A short seller deposits into an escrow account, the proceeds of the sale plus an additional amount of collateral called a haircut. The short seller earns interest on this money at a rate called the short rebate.

**Lease Rate:**

In general, the lease rate is the annualized rate required to borrow an asset. Equivalently, the lease rate is the annualized payment received in exchange for lending an asset.

In the case of stock, the lease rate is the rate of dividends; the short seller has to pay the person from whom he borrowed the stock the dividends. In the case of borrowing a commodity such as gold, the borrower may have to pay a lease rate to the person from whom the gold was borrowed. This would be an additional cost to the short seller.
Problems:

2.1 (2 points) On April 1, Martin short sold 1000 shares of stock for 6 months. The stock does not pay dividends. The bid and ask prices per share of the stock are:

<table>
<thead>
<tr>
<th>Date</th>
<th>Bid Price</th>
<th>Ask Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 1</td>
<td>178</td>
<td>179</td>
</tr>
<tr>
<td>October</td>
<td>170</td>
<td>171</td>
</tr>
</tbody>
</table>

A 0.3% commission was charged to engage in the short sale and a 0.3% commission to close the short sale. The continuously compounded risk-free interest rate is 3%. Calculate Martin’s profit.

A. Less than 7000
B. At least 7000, but less than 8000
C. At least 8000, but less than 9000
D. At least 9000, but less than 10,000
E. At least 10,000

2.2 (2 points) Define and briefly discuss the bid-ask spread.

2.3 (1 point) Give an example of a diversifiable risk. Give an example of an undiversifiable risk.

2.4 (2 points) The bid and ask prices for two stocks are as follows:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>110.25</td>
<td>110.75</td>
</tr>
<tr>
<td>XYZ</td>
<td>48.00</td>
<td>48.25</td>
</tr>
</tbody>
</table>

The broker’s commission is 0.4%.

(a) Anderson buys 100 shares of ABC stock. What is his cost?
(b) Xavier sells 500 shares of XYZ stock. What are his net proceeds from the sale?

2.5 (2 points) Define and briefly discuss reinsurance.

2.6 (2 points) The bid price of a stock is $88.25 per share, while the ask price is $88.75 per share. Harvey buys 100 shares of the stock, and pays commissions of 1%. Harvey turns around and immediately sells his shares, and pays commissions of 1%. What are his round trip transaction costs?

2.7 (1 point) Briefly define a haircut with respect to short selling.
2.8 (2 points) The ask price is $48.00 per share. The bid-ask spread is $0.25 per share. Ricardo buys 1000 shares of the stock, and pays commissions of 0.8%. Ricardo turns around and immediately sells his shares, and pays commissions of 0.8%. What are his round trip transaction costs?

2.9 (2 points) Define and briefly discuss catastrophe bonds.

2.10 (2 points) The bid and ask prices for a non-dividend paying stock are as follows:

<table>
<thead>
<tr>
<th>Date</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1</td>
<td>126.25</td>
<td>127.00</td>
</tr>
<tr>
<td>July 1</td>
<td>133.50</td>
<td>134.25</td>
</tr>
</tbody>
</table>

The broker’s commission is 0.5%.
You buy 100 shares on January 1 and sell them on July 1. Ignoring interest, what is your profit?

2.11 (1 point) Briefly define the short rebate with respect to short selling.

2.12 (2 points) The bid prices for a non-dividend paying stock are as follows:

<table>
<thead>
<tr>
<th>Date</th>
<th>Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 1</td>
<td>77.75</td>
</tr>
<tr>
<td>November 1</td>
<td>74.50</td>
</tr>
</tbody>
</table>

The bid-ask spread is 0.50.
The broker’s commission is 0.3%.
You short sell 200 shares on March 1 and close out your short position on November 1. Ignoring interest, what is your profit?

2.13 (1 point) List three reasons for short selling.
2.14 (3 points) The bid and ask prices for a non-dividend paying stock are as follows:

<table>
<thead>
<tr>
<th>Date</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 1</td>
<td>98.00</td>
<td>98.50</td>
</tr>
<tr>
<td>October 1</td>
<td>87.25</td>
<td>87.75</td>
</tr>
</tbody>
</table>

Tammy pays commissions of 0.6% when she buys and sells stock.
On April 1, Tammy short sells 400 shares of stock.
The proceeds of her short sale plus a haircut of 25% are deposited into an escrow account.
Her escrow account is credited with interest at a continuously compounded annual rate of 3%; in other words, the short rebate is 3%.
The continuously compounded annual risk free rate is 5%.
Tammy closes her short position on October 1.
What is Tammy’s profit?
A. 3500  B. 3700  C. 3900  D. 4100  E. 4300

2.15 (1 point) Give one reason for the introduction and use of derivatives in the 1970s.

2.16 (MFE Sample Introductory Q.25) Determine which of the following statements concerning risk sharing, in the context of financial risk management, is LEAST accurate.
(A) In an insurance market, individuals that do not incur losses have shared risk with individuals that do incur losses.
(B) Insurance companies can share risk by ceding some of the excess risk from large claims to reinsurers.
(C) Reinsurance companies can further share risk by investing in catastrophe bonds.
(D) Risk sharing reduces diversifiable risk, more so than reducing non-diversifiable risk.
(E) Ideally, any risk-sharing mechanism should benefit all parties sharing the risk.
Solutions to Problems:

2.1. **C.** In April, Martin borrows 1000 shares and sells them for: \((1000)(178) = 178,000\).
He pays commission of: \((0.3\%)(178,000) = 534\). He invests at 3\%: \(178,000 - 534 = 177,466\).
In October, Martin has \(177,466 e^{(1/2)(0.03)} = 180,148\).
Martin buys 1000 shares of stock and pays including commission:
\((1000)(171)(1.003) = 171,513\).
Thus Martin’s profit is: \(180,148 - 171,513 = 8635\).

2.2. The price at which you can buy is the ask (offer) price.
The lower price at which you can sell is the bid price.
The bid-ask spread is the difference, which allows the market maker in a stock to earn a living.

2.3. Diversifiable risks: slumping sales of a particular firm, a particular home catching on fire, the death of key employee of a firm, etc.
Undiversifiable risks: stock market crash, inflation, war, fluctuating interest rates, etc.
Comment: There are other possible answers.
Financial markets allow diversifiable risks to be shared, and undiversifiable risk to held by those most willing to do so. The existence of these risk sharing mechanisms benefits everyone.

2.4. (a) He pays the ask price: \((100)(110.75) = 11,075\).
Commissions are: \((0.4\%)(11,075) = 44.3\). Total cost is: \(11,075 + 44.3 = 11,119.30\).
(b) He receives the bid price: \((500)(48) = 24,000\). Commissions are: \((0.4\%)(24,000) = 96\).
Net proceeds is: \(24,000 - 96 = 23,904\).

2.5. Insurance companies buy from reinsurers insurance protection, which is called reinsurance.
For example, an insurer might buy catastrophic reinsurance in order to protect it from having to pay a lot of claims in the event of a large earthquake. Reinsurers pool many types of risks. Reinsurance helps to protect the solvency of insurers and allows the insurance market to operate.

2.6. Harvey pays: \((100)(88.75) = $8875\). Commissions are 88.75.
Harvey sells his shares for: \((100)(88.25) = $8825\). Commissions are 88.25.
Round trip transaction costs are: \(88.75 + 88.25 + 8875 - 8825 = $227\).

2.7. The collateral over and above the market value of the asset, required by the lender when an asset is borrowed.

2.8. Ricardo pays: \((1000)(48) = $48,000\). Commissions are: \((0.8\%)(48,000) = 384\).
Ricardo sells his shares for: \((1000)(47.75) = $47,750\). Commissions are: \((0.8\%)(47,750) = 382\).
Round trip transaction costs are: \(384 + 382 + 48,000 - 47,750 = $1016\).
2.9. A reinsurer (or large insurer) can issue catastrophe bonds, which the reinsurer need not repay if there is a specified catastrophic event (earthquake, hurricane, etc.) The bondholders receive higher interest payments in exchange for the risk of losing their investment in the event of the specified catastrophe. In the event of the specified catastrophe, the reinsurer can use the money from the bonds to help pay the resulting claims.

2.10. Pay the ask price: \((100)(127.00) = 12,700\).
Commissions are: \((0.5\%)(12,700) = 63.5\).
Receive the bid price: \((100)(133.50) = 13,350\).
Commissions are: \((0.5\%)(13,350) = 66.75\).
Profit is: \(13,350 \cdot 66.75 - 12,700 - 63.5 = 519.75\).

2.11. The rate of return on collateral when an asset is borrowed.

2.12. Sell shares, get the bid price: \((200)(77.75) = 15,550\).
Commissions are: \((0.3\%)(15,550) = 46.65\).
Buy shares, pay the ask price: \((200)(75.00) = 15,000\).
Commissions are: \((0.3\%)(13,350) = 45\).
Profit is: \(15,550 - 15,000 - 46.65 - 45 = 458.35\).

2.13. There are three reasons to short sell: speculation, financing, and hedging.

Her haircut is: \((25\%)(39,200) = 9800\).
Tammy deposits 39,200 + 9800 = 49,000 in the escrow account.
She pays a commission of: \((0.6\%)(39,200) = 235.20\).
In six months she buys 400 shares of stock for: \((400)(87.75) = 35,100\).
She pays a commission of: \((0.6\%)(35,100) = 210.60\).
Tammy gets the money in the escrow account plus interest: \(49,000 e^{0.03/2} = 49,740.54\).
Her initial costs are the haircut of 9800 and commission of 235.20.
Her profit is: \(49,740.54 - 35,100 - 210.60 - (9800 + 235.20)e^{0.05/2} = 4140.70\).

2.15. An increase in the price risk in several markets.
Comment: See page 6 and Figure 1.1 in Derivative Markets by McDonald.
2.16. C.
(A) is accurate because both types of individuals are involved in the risk-sharing process.
(B) is accurate because this is the primary reason reinsurance companies exist.
(C) is not accurate because reinsurance companies share risk by issuing rather than investing in catastrophe bonds. Reinsurers are ceding this excess risk to the bondholder.
(D) is accurate because it is diversifiable risk that is reduced or eliminated when risks are shared.
(E) is accurate because this is a fundamental idea underlying risk management and derivatives.
Comment: See Page 11 of Derivative Markets by McDonald.
An example of a diversifiable risk would be whether your car was in an accident. Whether your car is involved in an accident is (largely) uncorrelated with whether the other insureds have an accident. An example of a undiversifiable risk is a stock market crash.
Section 3, European Call Options

There are various types of options. The simplest and the most important type for this exam are European Options. The adjective “European” does not refer to where the option is bought.

Call Options:

ABC Stock is currently selling for $100. Dick buys from Jane an option to buy one year from today a share of ABC Stock for $150. If one year from now ABC Stock has a market price of more than $150 dollars, then Dick should use this option to buy a share of ABC Stock from Jane at $150. Dick could then sell this share of ABC Stock for the market price and make a profit.

This is an example of a European Call Option. A European Call Option gives the buyer the right to buy one share of a certain stock at a strike price (exercise price) upon expiration. A European option may only be exercised on one specific day. A call is an option to buy.

Dick has purchased a 1 year European Call Option on ABC Stock, with a strike price of $150.

Terminology:

The purchaser is the one who bought the call option. A purchased call is a call that has been bought; a long position in a call. Dick has a purchased call.

The writer is the one who sold the call (option). A written call is a call that has been sold; a short position in a call. Jane has a written call. Options are traded on an exchange. The writer of an option has to post margin with the exchange in order to guarantee they will fulfill their obligation if the buyer of the option decides to exercise it.

The strike price (exercise price) of the call is the amount that can be exchanged for the stock, or more generally the asset underlying the call option.

The call is exercised by its owner if the owner uses his option to exchange the strike price for the stock, or more generally the asset underlying the call option.

The expiration date in general is the last day on which an option can be exercised. European options can only be exercised at expiration.

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46 While for simplicity I have used in the example one share, one could buy an option for 100 shares or 1000 shares.

47 There are other exercise styles. See page 32 of Derivatives Markets by McDonald. Others will be discussed subsequently.
Payoff on a Call Option:

The eventual value to Dick of this option, depends on the price of ABC Stock one year from now.

For example, if ABC’s market price turned out to be $180 per share one year from today, then Dick could buy a share of ABC from Jane for the $150 strike price, and turn around and sell that share for $180. Dick would make a profit of $30, ignoring what he originally paid Jane to buy the option.\(^48\)

If the future price of ABC is $150 or less, then Dick would not exercise his option.\(^49\) In that case, his option turns out to have no value to Dick.

<table>
<thead>
<tr>
<th>Future Price of ABC</th>
<th>Payoff on the Option to Dick</th>
</tr>
</thead>
<tbody>
<tr>
<td>$120</td>
<td>0</td>
</tr>
<tr>
<td>$140</td>
<td>0</td>
</tr>
<tr>
<td>$160</td>
<td>$10</td>
</tr>
<tr>
<td>$180</td>
<td>$30</td>
</tr>
</tbody>
</table>

Dick is hoping that the future stock price will be higher than the strike, the higher the better for him. The purchaser of a call option hopes that the stock price at expiration will be high.

When the stock price is greater than the strike price, exercising the option makes money; this call option is “in-the-money.”

If the stock price and strike price are equal, then the option is “at-the-money.”

If the stock price is less than the strike price, then the call option is “out-of-the-money.”

Let \( Y_+ = \text{Max}[0, Y] = \begin{cases} Y & \text{if } Y \geq 0 \\ 0 & \text{if } Y < 0 \end{cases} \).

Then the eventual payoff on Dick’s call option is \((S_1 - 150)_+\), where \(S_1\) is the price of ABC Stock one year from now.

\(^48\) And ignoring any transaction costs.

\(^49\) Dick has not agreed to buy a share from Jane. Dick does not have an obligation to buy, rather Dick has purchased the right to buy a share if he wishes to.

\(^50\) This very useful actuarial notation is not on the syllabus of this exam.
Here is a graph of the payoff of Dick's (purchased) call option:

In general, the payoff of a European call option is: \((S_T - K)^+\), where \(S_T\) is the price of the stock on the expiration date of the call and \(K\) is the strike price of the call.

The payoff on a written call is minus that on a purchased call.
In the example, if Dick makes money then Jane who sold the call loses money and vice-versa.
Here is a graph of the payoff of Jane's written call option:

Jane is hoping that the future stock price will be lower than the 150 strike price.
The writer of a call option hopes that the stock price at expiration will be lower than the strike.
Profit on a Call:

The payoff on a call ignores the fact that the purchaser had to pay money to buy the call in the first place. **This purchase price is called the option premium.** In the example, let us assume that Dick paid $10 for his 1-year $150 strike call on ABC stock; the option premium is $10.51

We would also want to take into account the time value of money. Assume that the continuously compounded risk free rate is 5%. Then the future value of $10 option premium at expiration in one year is: $10 \cdot e^{0.05} = $10.51.52

In the example, Dick’s profit is defined as the payoff on his call minus 10.51.53

Here is a graph of Dick’s profit on his call as a function of the stock price at expiration:

![Graph of Dick's profit on his call](image)

This profit diagram is just the payoff diagram minus 10.51, the future value of the option premium.

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51 The price of the call would depend on among other things the current price of ABC stock. Pricing options will be discussed in subsequent sections.

52 In other words, the value accumulated for in this case one year of interest.

53 This profit is measured at the time of expiration rather than at the time of purchase of the option.
In general, **Profit = Payoff - Future Value of Option Premium.**

**Profit on a (purchased) call = \((S_T - K)_+ - C e^{rT},\)**

where \(C\) is the option premium, \(K\) is the strike price, and \(T\) is the time until expiration.

Exercise: At what stock price at expiration is Dick’s profit zero?
[Solution: \(150 + 10.51 = 160.41.\)]

If \(S_1 = 160.51\), then Dick’s profit is: \((160.51 - 150)_+ - 10 e^{0.05} = 0.\)

The profit on a written call is minus that on a purchased call.

**Profit on a written call = \(C e^{rT} - (S_T - K)_+\).**

The profits of the buyer and seller of a given call option sum to zero.

**Underlying Assets:**

A call gives the owner an option to buy an underlying asset. In the example, the underlying asset was a share of stock. However, we can also have a call on a stock index such as the Standard and Poor’s 500. We can have a call on currency such as Euros or Yen. We could have a call on a commodity such as oil, silver, or wheat, but as will be discussed subsequently, it is common to have instead calls on a futures contract on that commodity.
Insuring a Short Position, Caps.\textsuperscript{54}

Jesse short sells a share of XYZ stock. The current price of the stock is $100 and the stock pays no dividends. Jesse has agreed to return the share of stock in one year. Jesse sells the share for 100. He is concerned that the price of XYZ stock might go up.

Jesse buys a one-year 100-strike European call on XYZ stock.\textsuperscript{55} If in one year the price of the stock is more than $100, then Jesse will use his call to buy a share of stock stock for $100. If the price of the stock one year from now is at most $100, then Jesse does not use his call.

In any case, he pays no more than 100 to return the stock. He has placed a cap on his loss. His payoff is: $(S - 100)_+ - S$. Here is a graph of his payoff:

![Graph of Jesse's payoff]

Cap: Short stock and long call; a position that caps the maximum loss.

Buying the call provides Jesse with a type of insurance protection.

Assume that the call costs Jesse $14, and that the continuously compounded annual risk free rate is 5\%. Then his profit is: $(S_1 - 100)_+ + (100 - 14) e^{0.05} - S_1$.

\textsuperscript{54} See Section 3.1 of Derivative Markets by McDonald.

\textsuperscript{55} The call need not be at the money.
Exercise: What is Jesse’s profit, if the stock price in one year is $80?
[Solution: $0 + 90.41 - 80 = 10.41.$

Comment: If he had not bought the call then his profit would have been: $100e^{0.05} - 80 = 25.13.$]

Exercise: What is Jesse’s profit, if the stock price in one year is $130?
[Solution: $30 + 90.41 - 130 = -9.59.$

Comment: If he had not bought the call, then his profit would have been: $100e^{0.05} - 130 = -24.87.$]

Here is a graph of his profit as a function of the stock price at expiration:

![Graph](image)

**Writing Covered Calls:**

Emma has bought a share of XYZ stock. The current price of the stock is $100 and the stock pays no dividends. She is concerned that the price of her stock might go down.

Emma writes (sells) a 100-strike European call on the stock.\(^{56}\) Let us assume the call is one year. If in one year the price of the stock is more than $100, then the person to whom Emma sold the call will use it to buy stock from Emma for $100. If the price of the stock one year from now is at most $100, then the person to whom Emma sold the call does not use it.

Her payoff is: $S - (S-100)_+.$

\(^{56}\) The call need not be at the money.
Here is a graph of her payoff:

![Graph of Payoff]

**Covered Call: Long stock and short call.**
To be distinguished from writing a naked call, where one does not have a long position in the underlying asset.

While Jesse can be thought of as buying a type of insurance protection, Emma has done the opposite and can be thought of as selling insurance protection.

Assume that the call costs $14, and that the continuously compounded annual risk free rate is 5%.

Then her profit is: \( S - (S-100)^+ - (100 - 14) e^{0.05} \). Here is a graph of her profit:

![Graph of Profit]
3.1 (2 points) Graph the payoff of a European call option with a strike price of 100, as a function of the future stock price.

3.2 (3 points) Robert buys 1000 calls on a stock with a strike price of $120. The premium per call is $8. Robert also pays a total commission of $100. Determine the stock price at expiration at which Robert will break even. Graph Robert's profit as a percent of his initial investment, as a function of the stock price at expiration of the call. (Ignore the time value of money.)

3.3 (2 points) An 8-month European call option has a strike price of 90, and a premium of 7. The continuously compounded risk free rate is 6%. Graph the profit as a function of the future stock price.

3.4 (2 points) A 6-month European call option has a strike price of 50, and a premium of 4. The continuously compounded risk free rate is 8%. Graph the profit of a written call as a function of the future stock price.

3.5 (1 point) A 2-year European call option has a strike price of 70, and a premium of 9. The continuously compounded risk free rate is 4%. At what stock price in two years is the profit zero?

3.6 (2 points) Brandon buys a 4-month $60-strike call on a stock. The premium per call is $5. The risk free continuously compounded annual interest rate is 10%.
   (a) Determine Brandon’s profit if the spot price of the stock at expiration is 70.
   (b) Determine Brandon’s profit if the spot price of the stock at expiration is 50.

3.7 (2 points) Graph the payoff on a European call option with a strike price of 70, as a function of the future stock price.

3.8 (2 points) Rachel purchases an 1-year call on a stock. The call premium is $13. The continuously compounded annual interest rate is 6%. If the price of the stock at expiration is $103, then Rachel makes a profit of $4.20. Determine the strike price of the call.

3.9 (2 points) Daniel writes a 6-month $120-strike European call for $13. The continuously compounded annual interest rate is 8% per year. What is the range of Daniel's possible profit?
3.10 (3 points) Gilbert sells a share of a nongdividend paying stock for $67.
Gilbert also buys a $70-strike 1-year European call on this stock for a premium of $8.
Gilbert sells his share of stock in one year.
Sullivan buys a share of the same nongdividend paying stock, and also sells his share in one year.
The continuously compounded annual interest rate is 7% per year.
For what values of price of the stock in one year is Gilbert’s profit greater than Sullivan’s?

3.11 (2 points) Elizabeth writes an 9-month $130-strike call on a stock.
The call premium is $8.
The risk free continuously compounded annual interest rate is 9%.
(a) Elizabeth’s profit if the spot price of the stock at expiration is 150.
(b) Elizabeth’s profit if the spot price of the stock at expiration is 110.

3.12 (2 points) Logan short sells a share of nongdividend paying stock for $146.
Logan also buys a $150-strike 9-month European call on this stock for a premium of $16.
Logan buys a share of stock in nine months.
The continuously compounded annual interest rate is 5% per year.
Graph the Logan’s profit as a function of the stock price in nine months.

3.13 (2 points) Savannah buys a nongdividend paying stock index whose current price 817.
Savannah also writes (sells) a 800-strike one-year European call on this stock index for a premium of 111.
Savannah will sell the stock index in one year.
The continuously compounded annual interest rate is 6% per year.
Graph the Savannah’s profit as a function of the stock index price in one year.
3.14 (2 points) An investor purchases a dividend-paying stock and writes a t-year, European call option for this stock, with call premium C.
The stock pays dividends at a continuous at a rate $\delta$.
The stock price at time of purchase is $S_0$ and strike price are both $K$.
Assume that there are no transaction costs.
The risk-free annual force of interest is a constant $r$.
Let $S$ represent the stock price at time $t$.
$S_t > K$.
Determine an algebraic expression for the investor’s profit at expiration.

3.15 (3 points) Hansel buys 100 shares of a nondividend paying stock for $91 per share.
Hansel writes 100 $90-strike 1-year European calls on this stock for a premium of $11 per call.
Hansel sells his 100 shares of stock in one year.
Gretel buys 100 shares of the same stock, and also sells her shares in one year.
The continuously compounded annual interest rate is 6% per year.
For what values of price of the stock in one year is Hansel’s profit greater than Gretel’s?

3.16 (2 points) The current price of a stock is $60.
What would it mean for a call on that stock to be: in-the-money, at-the-money, or out-of-the-money?

3.17 (2 points) You buy a one-year 80-strike call for $9. The effective annual risk free rate is 5%.
What is the absolute value of your maximum possible loss?
What is your maximum possible gain?

3.18 (2 points) Graph the payoff on a written European call option with a strike price of 120, as a function of the future stock price.

3.19 (CAS5B, 5/95, Q.32) (2 points) The RegLuar Firm, Inc., a publicly held corporation, having current assets of $75 million and no liabilities, borrows $50 million by issuing a zero coupon bond due in two years. Assume no other transactions occur after the bond is issued and before it is redeemed.
a. (1/2 point) Briefly describe this transaction in terms of options.
b. (3/4 points) If the value of the company's assets falls to $40 million at the end of one year, discuss whether the stock has a nonzero value.
c. (3/4 points) At the end of two years, if the value of the company's assets falls to $40 million just before the debt is paid, discuss whether the stock has nonzero value.
3.20 (MFE Sample Introductory Q.11) Stock XYZ has the following characteristics:

- The current price is 40.
- The price of a 35-strike 1-year European call option is 9.12.
- The price of a 40-strike 1-year European call option is 6.22.
- The price of a 45-strike 1-year European call option is 4.08.

The annual effective risk-free interest rate is 8%.

Let S be the price of the stock one year from now.

All call positions being compared are long.

Determine the range for S such that the 45-strike call produce a higher profit than the 40-strike call, but a lower profit than the 35-strike call.

(A) S < 38.13
(B) 38.13 < S < 40.44
(C) 40.44 < S < 42.31
(D) S > 42.31
(E) The range is empty.

3.21 (MFE Sample Introductory Q.42) An investor purchases a non-dividend-paying stock and writes a t-year, European call option for this stock, with call premium C.

The stock price at time of purchase and strike price are both K.

Assume that there are no transaction costs.

The risk-free annual force of interest is a constant r.

Let S represent the stock price at time t.

S > K.

Determine an algebraic expression for the investor’s profit at expiration.

(A) Ce^{rt}
(B) C(1+rt) - S + K
(C) Ce^{rt} - S + K
(D) Ce^{rt} + K(1 - e^{rt})
(E) C(1+r)^t + K\{1 - (1+r)^t\}

3.22 (MFE Sample Introductory Q.47) An investor has written a covered call.

Determine which of the following represents the investor’s position.

(A) Short the call and short the stock
(B) Short the call and long the stock
(C) Short the call and no position on the stock
(D) Long the call and short the stock
(E) Long the call and long the stock
Solutions to Problems:

3.1. Graph of the payoff of a European call with strike price of 100, \((S - 100)^+\):

![Graph of the payoff of a European call with strike price of 100, \((S - 100)^+\)](image)

3.2. We want: 
\[ (1000)(S_T - 120)^+ = (1000)(8) + 100 = 8100. \]
\[ \Rightarrow S_T = S128.10. \]

If \( S_T < 120 \), the calls are worthless, and Robert’s profit is: -8100. This is equivalent to -100%.

If \( S_T > 120 \), Robert’s profit is: 
\[ (1000)(S_T - 120) - 8100 = 1000S_T - 128,100. \]

As a ratio to his initial investment of 8100, this is: 
\[ 0.12346S_T - 15.815. \]

Here is a graph of Robert’s profit as a percent of his initial investment as a function of the stock price at expiration:

![Graph of Robert’s profit as a percent of his initial investment as a function of the stock price at expiration](image)

Comment: Notice the large leverage when one invests in an option.
A small change in the stock price at expiration has a large effect on Robert’s profit.
3.3. The future value of the purchase price of the call is: $7 \exp[(2/3)(6\%)] = 7.29$. The profit is: $(S_{2/3} - 90)_+ - 7.29$.

3.4. The future value of the purchase price of the call is: $4 \exp[(1/2)(8\%)] = 4.16$. The profit on a written call is: $4.16 - (S_{1/2} - 50)_+$. 

3.5. $70 + 9 e^{(2)(4\%)} = 79.75$.

3.6. The future value of the option premium is: $5 \exp[(1/3)(10\%)] = $5.17
(a) $(70 - 60)_+ - 5.17 = $4.83$.
(b) $(50 - 60)_+ - 5.17 = -$5.17$. 
3.7. Graph of the payoff on a European call with strike price of 70, \((S - 70)_+:\)

![Graph of the payoff on a European call with strike price of 70](image)

3.8. \[4.20 = (103 - K)_+ - 13 e^{0.06}. \Rightarrow K = 85.\]

3.9. The future value of the option premium is: \(13 e^{(1/2)(8\%)} = 13.53.\)
Thus his profit is: \(13.53 - (S_{1/2} - 120)_+.\)
His maximum profit occurs when \(S_{1/2} \leq 120;\) his maximum profit is: \(13.53 - 0 = $13.53.\)
His minimum profit occurs when \(S_{1/2} = \infty;\) his maximum profit is: \(13.53 - \infty = -\infty.\)
Comment: On a purchased call, there is no (theoretical) limit on the possible profit.
Therefore, on a written call there is no (theoretical) limit on the possible loss.
3.10. Gilbert’s profit is: \((67 - 8)e^{0.07} - S + (S-70)_+ = 63.28 - S + (S-70)_+\).

Sullivan’s profit is: \(67e^{0.07} - S = 71.86 - S\).

For \(S \geq 70\), Gilbert’s profit is: \(63.28 - S + (S-70) = -6.72\).

Gilbert’s profit is equal to Sullivan’s when: \(71.86 - S = -6.72. \Rightarrow S = 78.58\).

Gilbert’s profit is greater than Sullivan’s when: \(S > 78.58\).

Comment: Gilbert has established a cap on his loss.

The graph of their profits:

3.11. The future value of the option premium is: \(8 \exp[(3/4)(9\%)] = \$8.56\)

(a) \(8.56 - (150 - 130)_+ = -\$11.44\).

(b) \(8.56 - (110 - 130)_+ = \$8.56\).
3.12. His profit is: \((146 - 16)e^{(0.75)(0.05)} - S + (S - 150)_+ = 134.97 - S + (S - 150)_+\).

The graph of his profit:

Comment: Logan has established a cap on his loss.
3.13. Her profit is: $S - (S - 800)_+ - (817-111) e^{0.06} = S - (S - 800)_+ - 749.66$.

For $S \leq 800$, the profit is: $S - 749.66$.

For $S \geq 800$, the profit is: $S - (S - 800) - 749.66 = 50.34$.

The graph of her profit:

Comment: Savannah has written a covered call.

Note that her profit graph has the same shape as that of selling a 800-strike one-year European put. Due to put-call parity to be discussed subsequently, the put premium is:

$P = C + PV[K] - PV[F] = 111 + 800e^{-0.06} - 817 = 47.41$.

Thus the profit from writing a put is: $47.71e^{0.06} - (800 - S)_+ = 50.34 - (800 - S)_+$.

For $S \leq 800$, the profit is: $S - 749.66$.

For $S \geq 800$, the profit is: 50.34.

This is equal to Savannah’s profit.

3.14. D. We pay $S_0$ for the stock and receive $C$ for writing the call.

The future value is: $(C - S_0) e^{rt}$.

At time $t$, one owns $e^{\delta t}$ shares. The stock is worth $S_t e^{\delta t}$.

Since $S_t > K$, the payoff on the written call to the one who bought it is: $S_t - K$.

At time $t$, our portfolio is worth: $S_t e^{\delta t} - (S_t - K) = S_t (e^{\delta t} - 1) + K$.

Our profit is: $S_t (e^{\delta t} - 1) + K + (C - S_0) e^{rt}$.

Comment: Similar to MFE Sample Introductory Q.42.

The force of interest is the actuarial term for the continuously compounded rate of interest.
3.15. Hansel’s profit is: $100S - 100(S-90)_+ - (100)(91 - 11)e^{0.06} = 100S - 100(S-90)_+ - 8495$.

Gretel’s profit is: $100S - (100)(91)e^{0.06} = 100S - 9663$.

For $S > 90$, Hansel’s profit is: $100S - 100(S-90) - 8495 = 505$.

Hansel’s profit is equal to Gretel’s when: $100S - 9663 = 505$. $\Rightarrow S = 101.68$.

Hansel’s profit is greater than Gretel’s when: $S > 101.68$.

Comment: Hansel has written a covered call.

The graph of their profits:

3.16. The call would be at-the-money if its strike price is $60$.

The call would be out-of-the-money if its strike price were greater than $60$.

The call would be in-the-money if its strike price were less than $60$; if the call could be exercised today, then there would be a positive payoff.

3.17. If the stock price at expiration is less than $80$, then the call expires worthless.

Thus the absolute value of the maximum loss is the future value of the option premium:

$(1.05)(9) = 9.45$.

As the stock price approaches infinity, the payoff on your call is unlimited.

Thus the maximum possible gain is unlimited.

Comment: On a purchased call, there is no (theoretical) limit on the possible profit.

Therefore, on a written call there is no (theoretical) limit on the possible loss.
3.18. Graph of the payoff on a written European call with strike price of 120, -(S - 120)^+:

3.19. a. When a firm borrows, the equity holders exchange their claim on the assets of the firm for a call option on the whole firm with an strike price equal to the maturity value of the debt. In other words, the option holder can either exercise the call option at the exercise date, pay the strike price, and obtain the stock, or the option holder can choose not to exercise the call option and he or she is left with no stock. Similarly, the equity holders can either repay the debt at the maturity date for the par value and retain the assets of the corporation, or they can default on the debt, in which case they are left with nothing since the bondholders take the assets of the corporation.

b. There is still a chance that the asset of the company will increase beyond $50 million by the end of the second year, so the stock still has value. This is equivalent to an out-of-the money call with a year until expiration; such a call has a positive if small value.

c. The company’s assets are less than the money owed to the bondholders, so the stock is now worthless. An out-of-the money call at expiration is worthless.
3.20. C. The profit on the 35-strike call is: \((S-35)_+ - (9.12)(1.08) = (S-35)_+ - 9.85\).

The profit on the 40-strike call is: \((S-40)_+ - (6.22)(1.08) = (S-40)_+ - 6.72\).

The profit on the 45-strike call is: \((S-45)_+ - (4.08)(1.08) = (S-45)_+ - 4.41\).

For \(S \leq 35\), all the calls expire worthless, and the condition is not met.

For \(35 < S \leq 40\), the 45-strike call has a profit of -4.41, while the 35-strike call has a profit of at most \(5 - 9.85 = -4.85\). Thus the profit for the 35-strike call is not more than that for the 45-strike call.

For \(S \geq 45\), the profit for the 40-strike call is \(S - 46.72\), which is greater than the profit for the 45-strike call of \(S - 49.41\).

For \(40 < S < 45\), set the profits for the 35-strike and 45-strike calls equal:
\[(S-35) - 9.85 = 0 - 4.41. \Rightarrow S = 40.44. \text{ We are okay for } S > 40.44.\]

For \(40 < S < 45\), set the profits for the 40-strike and 45-strike calls equal:
\[(S-40) - 9.85 = 0 - 4.41. \Rightarrow S = 42.31. \text{ We are okay for } S < 42.31.\]

Thus the desired range is: \(40.44 < S < 42.31\).

Comment: Here is a graph of the profits:
3.21. D. We pay K for the stock and receive C for writing the call.
The future value is: \((C - K) e^{rt}\).
At time t, the stock is worth S, and since \(S > K\), the payoff on the written call to the one who bought it is: \(S - K\).
At time t, our portfolio is worth: \(S - (S-K) = K\).
Our profit is: \(K + (C - K) e^{rt} = C e^{rt} + K(1 - e^{rt})\).
Comment: The force of interest is the actuarial term for the continuously compounded rate of interest.

3.22. B. Writing a covered call involves taking a long position in an asset together with a written call on the same asset.
Comment: See page 66 of Derivative Markets by McDonald.
Writing a covered put involves taking a short position in an asset together with a written put on the same asset.
Section 4, European Put Options

Puts are the other basic type of options besides calls.

**Put Options:**

XYZ Stock is currently selling for $200. Mary buys from Rob an option to sell one year from today a share of XYZ Stock for $250. If one year from now XYZ Stock has a market price of less than $250, then Mary should buy a share of XYZ Stock at the market price and then use her option to sell a share of XYZ Stock to Rob for $250, making a profit.

This is an example of a European Put Option.
**A European Put Option gives the buyer the right to sell one share of a certain stock at a strike price (exercise price) upon expiration. A put is an option to sell.**

Mary has purchased a 1 year European Put Option on XYZ Stock, with a strike price of $250.

**Payoff on a Put Option:**

The eventual payoff to Mary of her option, depends on the price of XYZ Stock one year from now.

For example, if XYZ market price turned out to be $220 per share one year from today, then Mary could buy a share of XYZ for $220 and turn around and sell that share for $250 to Rob. Mary would make a profit of $30, ignoring what she originally paid Rob to buy the option.\(^57\)

If the future price of XYZ Stock is $250 or more, then Mary would not exercise her option.\(^58\) In this case, her option turns out to have no value to Mary.

<table>
<thead>
<tr>
<th>Future Price of XYZ</th>
<th>Payoff on the Option to Mary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$220</td>
<td>$30</td>
</tr>
<tr>
<td>$240</td>
<td>$10</td>
</tr>
<tr>
<td>$260</td>
<td>0</td>
</tr>
<tr>
<td>$280</td>
<td>0</td>
</tr>
</tbody>
</table>

Mary is hoping that the future stock price will be lower than the strike, the lower the better for her. **The purchaser of a put option hopes that the stock price at expiration will be low.**

\(^57\) And ignoring any transaction costs.

\(^58\) Mary has not agreed to sell a share to Rob. Mary does not have an obligation to sell, rather Mary has purchased the right to sell a share if she wishes to.
When the stock price is lower than the strike price, exercising the option makes money; this put option is “in the money”. If the stock price and strike price are equal, then the option is “at the money.” If the stock price is more than the strike price, then the put option is “out of the money.”

The eventual value of Mary’s put option is \((250 - S_1)_+\), where \(S_1\) is the price of XYZ Stock one year from now.\(^{59}\) Here is a graph of the payoff of Mary’s put option:

![Graph of the payoff of Mary’s put option](image)

In general, the payoff of a European put option is: \((K - S_T)_+\), where \(S_T\) is the price of the stock on the expiration date and \(K\) is the strike price.

\(^{59}\) \(Y_+\) is 0 if \(Y < 0\), and \(Y\) is \(Y \geq 0\).
The payoff on a written put is minus that on a purchased put. In the example, if Mary makes money then Rob who sold the put loses money and vice-versa. Here is a graph of the payoff of Rob’s written call option:

Rob is hoping that the future stock price will be higher than the 250 strike price. The writer of a put option hopes that the stock price at expiration will be higher than the strike.
The payoff on a put ignores the fact that the purchaser had to pay money to buy the call in the first place. This purchase price is called the option premium. In the example, Mary bought from Rob a one-year 250-strike put on XYZ Stock. Let us assume that the option premium is $30.\textsuperscript{60}

We would also want to take into account the time value of money. Assume that the continuously compounded risk free rate is 5%. Then the future value of $30 option premium at expiration in one year is: $30 e^{0.05} = $31.54.\textsuperscript{61}

In the example, Mary’s profit is defined as the payoff on his call minus 31.54.\textsuperscript{62}

Here is a graph of Mary’s profit on his call as a function of the stock price at expiration:

This profit diagram is just the payoff diagram minus 31.54, the future value of the option premium.

\textsuperscript{60} The price of the put would depend on among other things the current price of ABC stock. Pricing options will be discussed in subsequent sections.

\textsuperscript{61} In other words, the value accumulated for in this case one year of interest.

\textsuperscript{62} This profit is measured at the time of expiration rather than at the time of purchase of the option.
In general, Profit = Payoff - Future Value of Option Premium.

Profit on a (purchased) put = \((K - S_T)_+ - Pe^{rT}\),

where \(P\) is the option premium, \(K\) is the strike price, and \(T\) is the time until expiration.

Exercise: At what stock price at expiration is Mary’s profit zero?

[Solution: 250 - 31.54 = 218.46.]

If \(S_1 = 218.46\), then Mary’s profit is: \((250 - 218.46)_+ - 30 e^{0.05} = 0\).]

The profit on a written put is minus that on a purchased put.

Profit on a written put = \(Pe^{rT} - (K - S_T)_+\).

The profits of the buyer and seller of a given put option sum to zero.
Expected Future Value of an Option:

The future payoff on a European call option is: $(S_T - K)_+$, where $S_T$ is the price of the stock on the expiration date and $K$ is the strike price. The future payoff on a European put option is: $(K - S_T)_+$. Of course, at the time one could purchase an option, one does not know the future price of the stock. The future price of the stock is a random variable. The expected value of the option can be obtained by averaging using the distribution of future stock prices.


If one knew the distribution of $S_T$, then:

$$E[(S_T - K)_+] = E[S_T - K | S_T > K] \cdot \text{Prob}[S_T > K] + (0) \cdot \text{Prob}[S_T \leq K] = (E[S_T | S_T > K] - K) \cdot \text{Prob}[S_T > K].$$

Similarly,

$$E[(K - S_T)_+] = (0) \cdot \text{Prob}[S_T > K] + E[K - S_T | S_T \leq K] \cdot \text{Prob}[S_T \leq K] = (K - E[S_T | S_T \leq K]) \cdot \text{Prob}[S_T \leq K].$$

As will be discussed in subsequent sections, this is the idea behind pricing options. However, is will be discussed the expected values need to be taken with respect to “risk neutral probabilities”.

Insurance Policy as a Put Option:

An insured buys a homeowners insurance policy on a house worth $400,000. In the event of damage to the house, the policy will pay: $(400,000 - \text{value of the house after the damage})_+$. This is the same as the payoff on a put with $K = 400,000$, and $S = \text{value of the house after the damage}$. Thus an insurance policy covering property is analogous to a put option.

Owning the house is a long position, and the homeowners insurance protection is analogous to the protection provided by a long put. However, it is important to note that the homeowners insurance policy is only providing protection against the value of your home going down due to damage due to fire, wind, earthquake, etc. Homeowners insurance provides no protection against the price of your home going down due to market conditions.

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63 It will be a while until we discuss pricing options via Binomial Trees and the Black-Scholes Formula.

64 See Section 2.5 of McDonald, not on the syllabus.

65 Many details of actual homeowners insurance are being ignored solely for illustrative purposes. For example, a homeowners insurance policy also provides liability coverage.
Insuring a Long Position, Floors

Kaitlyn buys a share of XYZ stock. The current price of the stock is $100 and the stock pays no dividends. She is concerned that the price of her stock might go down.

Kaitlyn buys a 100-strike European put on XYZ stock. Let us assume the put is one year. If in one year the price of the stock is less than $100, then Kaitlyn will use her put to sell her stock for $100. If the price of the stock one year from now is at least $100, then Kaitlyn does not use her put.

In any case, she ends up with her stock or $100 whichever is worth more. She has placed a floor on the value of her position one year from now.

Here is a graph of her payoff:

![Graph showing the payoff of Kaitlyn's position]

**Floor: Long stock and long put; a position that guarantees a minimum price.**

Buying the put provides Kaitlyn with a type of insurance protection.

Assume that the put costs Kaitlyn $8, and that the continuously compounded annual risk free rate is 5%. Then her profit including the cost of buying the stock is: $S_1 + (100 - S_1)^+ - 108 \cdot e^{0.05}$.

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66 See Section 3.1 of *Derivative Markets* by McDonald.
67 She could instead already own a share of stock, but not wish to sell the stock.
68 The put need not be at the money.
Exercise: What is Kaitlyn’s profit, if the stock price in one year is $90?
[Solution: $90 + 10 - 108e^{0.05} = -13.54$.

Comment: If she had not bought the put, then her profit would have been: $90 - 100e^{0.05} = -15.13$.

Exercise: What is Kaitlyn’s profit, if the stock price in one year is $120?
[Solution: $120 + 0 - 108e^{0.05} = 6.46$.

Comment: If she had not bought the put, then her profit would have been:
$120 - 100e^{0.05} = 14.87$.

Here is a graph of her profit as a function of the stock price at expiration:
Writing Covered Puts:

Noah has shorted a share of XYZ stock. The current price of the stock is $100 and the stock pays no dividends. He is concerned that the price of her stock might go up.

Noah writes (sells) a 100-strike European put on the stock. Let us assume the put is one year. If in one year the price of the stock is more than $100, then the person to whom Noah sold the put will use it to sell their stock to Noah for $100. If the price of the stock one year from now is at most $100, then the person to whom Noah sold the put does not use it; Noah buys a share of stock at the market price.

In any case, Noah pays at most $100 for the share of stock that he needs to return to the person he borrowed it from when he shorted the stock.

Here is a graph of his payoff:

Covered Put: Short stock and short put.

While Kaitlyn can be thought of as buying a type of insurance protection, Noah has done the opposite and can be thought of as selling insurance protection.

Assume that the put costs $8, and that the continuously compounded annual risk free rate is 5%. Then his profit is: $108 e^{0.05} - S_1 - (100 - S_1)^+.$

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69 The put need not be at the money.
70 Noah’s position and payoff is the opposite of Kaitlyn who bought a floor.
71 To be distinguished from writing a naked put, where one does not have a short position in the underlying asset.
Here is a graph of his profit as a function of the stock price at expiration:
Problems:

Use the following information for the next 3 questions:
The price of the stock of the Daily Planet Media Company 1 year from now has the following distribution:

<table>
<thead>
<tr>
<th>Price</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>20%</td>
</tr>
<tr>
<td>80</td>
<td>30%</td>
</tr>
<tr>
<td>100</td>
<td>30%</td>
</tr>
<tr>
<td>120</td>
<td>20%</td>
</tr>
</tbody>
</table>

4.1 (1 point) Determine the expected stock price of Daily Planet Media Company one year from now.

4.2 (1 point) Determine the expected payoff of a 1 year European call option on one share of Daily Planet Media Company, with a strike price of 85.
   (A) Less than 7
   (B) At least 7, but less than 9
   (C) At least 9, but less than 11
   (D) At least 11, but less than 13
   (E) At least 13

4.3 (1 point) Determine the expected payoff of a 1 year European put option on one share of Daily Planet Media Company, with a strike price of 85.
   A) Less than 7
   B) At least 7, but less than 9
   C) At least 9, but less than 11
   D) At least 11, but less than 13
   E) At least 13

4.4 (2 points) Graph the payoff of a European put option with a strike price of 100, as a function of the future stock price.

4.5 (3 points) Kimberly buys 100 puts on a stock with a strike price of $80. The premium per put is $5. Kimberly also pays a total commission of $60. Determine the stock price at expiration at which Kimberly will break even. Graph Kimberly’s profit as a percent of her initial investment, as a function of the stock price at expiration of the put. (Ignore the time value of money.)
4.6 (2 points) Let $S(t)$ be the price of a stock at time $t$.
The stock pays dividends at the continuously compounded rate $\delta$.
The continuously compounded risk free rate is $r$.
Assume a contract is purchased at time 0 and pays at time $T$: $\text{Max}[S(T), 100]$.
Determine the premium for this contract in terms of the premium of a European option and other known quantities.

4.7 (1 point) Several years ago Warren bought 1000 shares of XYZ stock.
Fortunately for Warren the value of XYZ stock has increased substantially since then.
However, for tax reasons Warren does not wish to sell his XYZ stock and realize his capital gains.
Rather Warren plans to sell his XYZ stock one year from now.
Warren is worried that by time he is ready to sell his stock his capital gains may decrease or vanish.
Briefly describe how Warren could purchase a European option to hedge this risk.

4.8 (1 point) The Rich and Fine Stock Index has a current price of 800.
An insurer offers a contract that will pay the value of the Rich and Fine Stock Index two years from now; however, the contract will pay a minimum of 750. The insurer buys the index.
Briefly describe how the insurer could purchase a European option to hedge its risk.

4.9 (1 point) Which of the following statements are true?
1. A European call option can be exercised on or before the expiration date.
2. A put option will be worthless at the expiration date if the share price at that time is less than the strike price.
3. An investor who sells a stock short sells something that he or she does not yet own.
A. 1 and 2 only.
B. 1 and 3 only.
C. 2 and 3 only.
D. 1, 2, and 3.
E None of A, B, C, or D is correct.

4.10 (2 points) Let $Y_+$ equal the maximum of $Y$ and zero.
Let $S$ and $Q$ be two random variables.
(a) Determine $(S - Q)_+ + (Q - S)_+$.
(b) Determine $(S - Q)_+ - (Q - S)_+$.

4.11 (2 points) Graph the payoff on a written European put option with a strike price of 110, as a function of the future stock price.

4.12 (2 points) An 18-month European put option has a strike price of 90, and a premium of 11.
The continuously compounded risk free rate is 3%.
Graph the profit of a purchased put as a function of the future stock price.
4.13 (2 points) Tyler writes an 1-year $100-strike put on a stock. The put premium is $12. The risk free continuously compounded annual interest rate is 6%.
(a) Tyler’s profit if the spot price of the stock at expiration is 120.
(b) Tyler’s profit if the spot price of the stock at expiration is 80.

4.14 (2 points) Which of the following statements is false?
A. A long put will provide “insurance protection” to the owner of an asset.
B. The maximum gain on a purchased call is unlimited
C. The maximum gain on a written put is the future value of the put premium.
D. A long call will provide “insurance protection” to someone with a short position in an asset.
E. None of A, B, C, or D

4.15 (3 points) Jennifer buys a 2-year 1200-strike European call option on a stock index for 203. Ryan writes a 2-year 1100-strike European put option on the same stock index for 122. The continuously compounded annual interest rate is 5% per year. For what values of the stock index at expiration is Jennifer’s profit more than Ryan’s?

4.16 (2 points) Alexander owns a share of stock. Alexander buys a $80-strike European put on this stock. Alexander holds the share of stock until expiration of the put. Graph the value of Alexander’s position at the time of expiration of the put as a function of the stock price at expiration.

4.17 (1 point) Which of the following have a potential for an unlimited loss.
A. Call B. Put C. Written Call D. Written Put E. None of A, B, C, or D

4.18 (2 points) Jessica buys a 6-month $70-strike put on a stock. The option premium is $7. The risk free continuously compounded annual interest rate is 10%.
(a) Determine Jessica’s profit if the spot price of the stock at expiration is 80.
(b) Determine Jessica’s profit if the spot price of the stock at expiration is 60.

4.19 (2 points) Natalie buys a share of nondividend paying stock for $73. Natalie also buys a $75-strike 6-month European put on this stock for a premium of $5. Natalie sells her share of stock in six months. The continuously compounded annual interest rate is 6% per year. Graph the Natalie’s profit as a function of the stock price in six months.

4.20 (1 point) A 2-year European put option has a strike price of 80, and a premium of 11. The continuously compounded risk free rate is 5%. At what stock price in two years is the profit zero?
4.21 (2 points) Caleb shorts a nondividend paying stock index whose current price is $1384. Caleb also writes (sells) a 1400-strike one-year European put on this stock index for a premium of $90. Caleb will buy the stock index in one year in order to close out his short position. The continuously compounded annual interest rate is 4% per year. Graph the Caleb’s profit as a function of the stock index price in one year.

4.22 (3 points) Chip short sells 100 shares of a nondividend paying stock for $118 per share. Chip also sells 100 $110-strike 1-year European puts on this stock for a premium of $13 per put. Chip buys 100 shares of stock in one year in order to close out his short position. Dale short sells 100 shares of the same stock, and also sells his shares in one year. The continuously compounded annual interest rate is 5% per year. For what values of price of the stock in one year is Chip’s profit greater than Dale’s?

4.23 (2 points) Madison buys 100 shares of Big Belly Burgers stock at $42 per share. Madison also buys 40-strike 3-month puts on 100 shares of Big Belly Burgers stock at $2 per put. The continuously compounded risk free rate is 10% per year. Calculate the smallest profit Madison can have in 3 months.
A. -700  B. -600  C. -500  D. -400  E. -300

4.24 (2 points) The current price of a stock is $80. What would it mean for a put on that stock to be: in-the-money, at-the-money, or out-of-the-money?

4.25 (2 points) Hannah buys an 8-month $90-strike European put for $11. The continuously compounded annual interest rate is 6% per year. What is the range of Hannah’s possible profit?

4.26 (3 points) Options on a stock has the following characteristics:
• The price of a 60-strike 2-year European put option is 8.24.
• The price of a 70-strike 2-year European put option is 12.85.
• The price of a 80-strike 2-year European put option is 18.30.
The annual continuously compounded risk-free interest rate is 6%.
Let S be the price of the stock two years from now. All put positions being compared are long. Determine the range for S such that the 80-strike put produces a higher profit than the 70-strike put, but a lower profit than the 60-strike put.
(A) S < 68.66
(B) 68.66 < S < 73.86
(C) 73.86 < S < 75.20
(D) S > 75.20
(E) The range is empty.
4.27 (1 point) You buy a 6-month 100-strike put for $8. 
The continuously compounded annual risk free rate is 6%. 
What is the absolute value of your maximum possible loss? 
What is your maximum possible gain?

4.28 (3 points) Sonny buys a share of a nondividend paying stock for $56. 
Sonny also buys a $60-strike 1-year European put on this stock for a premium of $9. 
Sonny sells his share of stock in one year. 
Cher buys a share of the same nondividend paying stock, and also sells her share in one year. 
The continuously compounded annual interest rate is 8% per year. 
For what values of price of the stock in one year is Sonny’s profit greater than Cher’s?

4.29 (2 points) You sell a 2-year 60-strike put for $7. 
The continuously compounded annual risk free rate is 5%. 
What is the absolute value of your maximum possible loss? 
What is your maximum possible gain?

4.30 (2 points) Graph the payoff on a European put option with a strike price of 60, as a function of 
the future stock price.

4.31 (2 points) Which of the following statements is false? 
A. The payoff diagram of a short put has the same shape as the payoff diagram of a covered call. 
B. The payoff diagram of a short call has the same shape as the payoff diagram of a covered put. 
C. The payoff diagram of a long call has the same shape as the payoff diagram of a floor. 
D. The payoff diagram of a long put has the same shape as the payoff diagram of a cap. 
E. None of A, B, C, or D

4.32 (2 points) A 9-month European put option has a strike price of 80, and a premium of 6. 
The continuously compounded risk free rate is 5%. 
Graph the profit of a written put as a function of the future stock price.

4.33 (2 points) Consider a European put option on a stock without dividends, with 18 months to 
expiration and a strike price of 120. 
Suppose that the effective annual interest rate is 5%, and that the put costs 15.80 today. 
Calculate the price that the stock must be in 18 months so that being long in the put would produce 
the same profit as being short in the put. 
(A) 102.2    (B) 102.4    (C) 102.6    (D) 102.8    (E) 103.0
4.34 (CAS5B, 5/95, Q.13) (1 point) You own a share of XYZ stock and are concerned that the price of the stock may fall. Of the following choices, which would allow you to offset (at least partially) potential future losses?
1. Buy a put on the share of stock.
2. Sell the stock short.
3. Sell a call on the share of stock.
A. 1 B. 2 C. 1, 2 D. 1, 2, 3 E. None of 1, 2, 3

4.35 (CAS5B, 5/95, Q.35) (1.5 points)
a. (1 point) Explain how a term life insurance policy on the life of an actively employed actuary can function similarly to a put option owned by the actuary's dependents.
b. (1/2 point) Under what circumstances does the term life policy fall short of operating like a put?

4.36 (MFE Sample Introductory Q.12) Consider a European put option on a stock index without dividends, with 6 months to expiration and a strike price of 1,000.
Suppose that the effective six-month interest rate is 2%, and that the put costs 74.20 today.
Calculate the price that the index must be in 6 months so that being long in the put would produce the same profit as being short in the put.
(A) 922.83 (B) 924.32 (C) 1,000.00 (D) 1,075.68 (E) 1,077.17

4.37 (MFE Sample Introductory Q.13) A trader shorts one share of a stock index for 50 and buys a 60-strike European call option on that stock that expires in 2 years for 10.
Assume the annual effective risk-free interest rate is 3%.
The stock index increases to 75 after 2 years.
Calculate the profit on your combined position, and determine an alternative name for this combined position.

<table>
<thead>
<tr>
<th>Profit</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) -22.64</td>
<td>Floor</td>
</tr>
<tr>
<td>(B) -17.56</td>
<td>Floor</td>
</tr>
<tr>
<td>(C) -22.64</td>
<td>Cap</td>
</tr>
<tr>
<td>(D) -17.56</td>
<td>Cap</td>
</tr>
<tr>
<td>(E) -22.64</td>
<td>“Written” Covered Call</td>
</tr>
</tbody>
</table>

4.38 (MFE Sample Introductory Q.35)
A customer buys a 50-strike put on an index when the market price of the index is also 50.
The premium for the put is 5.
Assume that the option contract is for an underlying 100 units of the index.
Calculate the customer’s profit if the index declines to 45 at expiration.
(A) -1000 (B) -500 (C) 0 (D) 500 (E) 1000
4.39 (MFE Sample Introductory Q.48) For a certain stock, Investor A purchases a 45-strike call option while Investor B purchases a 135-strike put option. Both options are European with the same expiration date. Assume that there are no transaction costs. If the final stock price at expiration is S, Investor A's payoff will be 12. Calculate Investor B's payoff at expiration, if the final stock price is S.
(A) 0  (B) 12  (C) 36  (D) 57  (E) 78

4.40 (MFE Sample Introductory Q.49) The market price of Stock A is 50. A customer buys a 50-strike put contract on Stock A for 500. The put contract is for 100 shares of A. Calculate the customer’s maximum possible loss.
(A) 0  (B) 5  (C) 50  (D) 500  (E) 5000

4.41 (MFE Sample Introductory Q.50) An investor bought a 70-strike European put option on an index with six months to expiration. The premium for this option was 1. The investor also wrote an 80-strike European put option on the same index with six months to expiration. The premium for this option was 8. The six-month interest rate is 0%. Calculate the index price at expiration that will allow the investor to break even.
(A) 63  (B) 73  (C) 77  (D) 80  (E) 87

4.42 (MFE Sample Introductory Q.61) An investor purchased Option A and Option B for a certain stock today, with strike prices 70 and 80, respectively. Both options are European one-year put options. Determine which statement is true about the moneyness of these options, based on a particular stock price.
(A) If Option A is in-the-money, then Option B is in-the-money.
(B) If Option A is at-the-money, then Option B is out-of-the-money.
(C) If Option A is in-the-money, then Option B is out-of-the-money.
(D) If Option A is out-of-the-money, then Option B is in-the-money.
(E) If Option A is out-of-the-money, then Option B is out-of-the-money.
4.43 (MFE Sample Introductory Q.62)  
The price of an asset will either rise by 25% or fall by 40% in 1 year, with equal probability. A European put option on this asset matures after 1 year.  
Assume the following:  
• Price of the asset today: 100 
• Strike price of the put option: 130 
• Put option premium: 7 
• Annual effective risk free rate: 3%  
Calculate the expected profit of the put option.  
(A) 12.79  (B) 15.89  (C) 22.69  (D) 27.79  (E) 30.29

4.44 (MFE Sample Introductory Q.66) The current price of a stock is 80. Both call and put options on this stock are available for purchase at a strike price of 65. Determine which of the following statements about these options is true.
(A) Both the call and put options are at-the-money.  
(B) Both the call and put options are in-the-money.  
(C) Both the call and put options are out-of-the-money.  
(D) The call option is in-the-money, but the put option is out-of-the-money.  
(E) The call option is out-of-the-money, but the put option is in-the-money.
Solutions to Problems:

4.1. \( (20\%)(60) + (30\%)(80) + (30\%)(100) + (20\%)(120) = 90. \)

4.2. D. \( (20\%)(0) + (30\%)(0) + (30\%)(100 - 85) + (20\%)(120 - 85) = 11.5. \)
   Comment: In order to price an option, one would use “risk neutral probabilities”.

4.3. A. \( (20\%)(85 - 60) + (30\%)(85 - 80) + (30\%)(0) + (20\%)(0) = 6.5. \)

4.4. Graph of the payoff of a European put with strike price of 100, \( (100 - S)_+ \):

![Graph of the payoff of a European put with strike price of 100](image)
4.5. We want: \((100)(80 - S_T)_+ = (100)(5) + 60 = 560. \implies S_T = $74.40$.

If \(S_T > 74.40\), the puts are worthless, and Kimberly’s profit is: -560. This is equivalent to -100%.

If \(S_T < 74.40\), Kimberly’s profit is: \((100)(80 - S_T)_+ - 560 = 7440 - 100S_T\).

As a ratio to her initial investment of 560, this is: \(13.2857 - 0.17857S_T\).

Here is a graph of Kimberly’s profit as a percent of her initial investment as a function of the stock price at expiration:

Comment: Notice the large leverage when one invests in an option.

A small change in the stock price at expiration has a large effect on Kimberly’s profit.

4.6. \(\text{Max}[S(T), 100] = 100 + \text{Max}[S(T) - 100, 0] = 100 + (S(T) - 100)_+\).

\((S(T) - 100)_+ \) is the payoff on a T-year 100-strike European call on this stock.

The present value of a payoff T years from now of 100 is: \(100 e^{-rT}\).

Thus the premium for this contract is: \(100 e^{-rT} + C\),

where \(C\) is the premium on a T-year 100-strike European call.

Alternately, \(\text{Max}[S(T), 100] = S(T) + \text{Max}[100 - S(T), 0] = S(T) + (100 - S(T))_+\).

\((100 - S(T))_+ \) is the payoff on a T-year 100-strike European put on this stock.

The prepaid forward price for \(S(T)\) is: \(S(0) e^{-\delta T}\)

Thus the premium for this contract is: \(S(0) e^{-\delta T} + P\),

where \(P\) is the premium on a T-year 100-strike European call.

Comment: As will be discussed, the call premium and put premiums are connected via put-call parity. Thus one can show that the two forms of the premium for this contract are equivalent.
4.7. Warren could buy one thousand 1-year at-the-money European puts on XYZ stock. If one year from now XYZ stock is worth more than its current price, then he can sell his stock and make more in capital gains than he has currently. If one year from now XYZ stock is worth less than its current price, then he could use his puts to sell his stock at its price today and make in capital gains the amount he has currently. 

Comment: Buying a put protects against the price of a stock you own going down.
Buying a call would protect against the price of a stock you shorted going up.

4.8. The insurer could buy a 2-year 750-strike European put on the Rich and Fine Stock Index. If two years from now the index is worth more than 750, then the insurer can sell the index, pay off the contract, and have some money left over.
If two years from now the index is worth less than 750, then the insurer can use its put to sell the index for 750 and pay off the contract.

4.9. E.
Statement number 1 is false. A European options can be exercised only on the expiration date. The statement is the correct definition for an American option.
Statement number 2 is false. In fact, the opposite is true: the lower the market price on the expiration date, the higher the value of the put option.
Statement number 3 is correct. Short sellers sell stock which they do not yet own.

4.10. (a) \((S - Q)_+ = \begin{cases} 
S - Q & \text{if } S \geq Q \\
0 & \text{if } S < Q
\end{cases} \quad \text{and} \quad (Q - S)_+ = \begin{cases} 
0 & \text{if } S \geq Q \\
Q - S & \text{if } S < Q
\end{cases}.

Therefore, \((S - Q)_+ + (Q - S)_+ = \begin{cases} 
S & \text{if } S \geq Q \\
Q - S & \text{if } S < Q
\end{cases} = |S - Q|.

(b) \((S - Q)_+ = \begin{cases} 
S - Q & \text{if } S \geq Q \\
0 & \text{if } S < Q
\end{cases} \quad \text{and} \quad (Q - S)_+ = \begin{cases} 
0 & \text{if } S \geq Q \\
Q - S & \text{if } S < Q
\end{cases}.

Therefore, \((S - Q)_+ - (Q - S)_+ = \begin{cases} 
S & \text{if } S \geq Q \\
S - Q & \text{if } S < Q
\end{cases} = S - Q.

Comment: If Q were a constant, then \((S - Q)_+ + (Q - S)_+\) would be the payoff on a Q-strike call and the similar Q-Strike put; in other words the payoff on a straddle is: \((S - K)_+ + (K - S)_+ = IS - QI.
If Q were a constant, then \((S - Q)_+ - (Q - S)_+\) would be the payoff on a Q-strike call and the sale of a similar Q-Strike put. The fact that \((S - K)_+ - (K - S)_+ = S - K\), is the basis of put-call parity, to be discussed in a subsequent section.
4.11. Graph of the payoff on a European put with strike price of 110, \(-(110 - S)_{+}\):

4.12. The future value of the purchase price of the put is: $11 \exp[(1.5)(3\%)] = 11.51$. The profit on a purchased put is: $(90 - S_{1.5})_{+} - 11.51$.

4.13. The future value of the option premium is: $12 \exp[(1)(6\%)] = $12.74.
(a) $12.74 - (100 - 120)_{+} = $12.74.
(b) $12.74 - (100 - 80)_{+} = -$7.26.
4.14. E. A floor is long stock and long put, and puts a floor on the payoff. Thus A is true.
The payoff on a call is \((S_T - K)_+\), which is unlimited. Thus B is true.
The profit is: \(\text{FV[put premium]} - (K - S_T)_+\). This is greatest for \(K < S_T\), when it is \(\text{FV[put premium]}\).
Thus C is true.
A cap is short stock and long call, and puts a cap on the potential loss. Thus D is true.

4.15. Jennifer's profit is: \((S_2 - 1200)_+ - 203 \exp[(2)(5\%)] = (S_2 - 1200)_+ - 224.\nRyan’s profit is: \(122 \exp[(2)(5\%)] - (1100 - S_2)_+ = 135 - (1100 - S_2)_+\).
For \(S_2 \leq 1200\), Jennifer's profit is -224.
For \(S_2 \geq 1100\), Ryan’s profit is 135.
Thus for \(1200 \geq S_2 \geq 1100\), Ryan’s profit is greater than Jennifer's.
For \(S_2 \leq 1100\), set Ryan’s profit equal to Jennifer’s: -224 = 135 - (1100 - S_2). \(\Rightarrow\) \(S_2 = 741\).
\(\Rightarrow\) Jennifer’s profit is more than Ryan’s for \(S_2 < 741\).
For \(S_2 \geq 1200\), set Ryan’s profit equal to Jennifer’s: 135 = \((S_2 - 1200) - 224. \(\Rightarrow\) \(S_2 = 1559\).
\(\Rightarrow\) Jennifer’s profit is more than Ryan’s for \(S_2 > 1559\).
In summary, Jennifer’s profit is more than Ryan’s for either \(S_2 < 741\) or \(S_2 > 1559\).
Comment: A graph of their profits:
4.16. The payoff on the put is \((80 - S)_+\)

Thus the value of his position is: 
\[
S + (80 - S)_+ = \begin{cases} 
S & \text{if } S > 80 \\
80 & \text{if } S \leq 80
\end{cases}
\]

The graph of the value of his position at expiration:

4.17. C. A put has a finite maximum payoff, while a call has no maximum payoff. Thus a written put has a maximum loss, while a written call has no maximum possible loss. 

Comment: Since buying a stock has no maximum profit, short selling a stock has a potential for an unlimited loss.

4.18. The future value of the option premium is: 
\[
7 \exp[(1/2)(10\%)] = 7.36
\]

(a) \((70 - 80)_+ - 7.36 = -7.36\).

(b) \((70 - 60)_+ - 7.36 = -2.64\).
4.19. The future value of the cost of the stock plus put is: \((73 + 5)e^{0.03} = 80.38\).
Her profit is: \(S + (75 - S)^+ - 80.38\).
The graph of her profit:

---

Comment: Natalie has established a floor on her profit.

4.20. \(80 - 11 e^{(2)(5\%)} = 67.84\).
4.21. The future value of the value of the money Caleb gets for selling the stock index plus put is:

\[(1384 + 90)e^{0.04} = 1534.\]

His profit is: \[1534 - S + (1400 - S)_+.\]

For \(S \leq 1400\), the profit is: 134.

For \(S \geq 1400\), the profit is: 1534 - S.

The graph of his profit:

Comment: Caleb has written a covered put.

Note that his profit graph has the same shape as that of selling a 1400-strike one-year European call. Due to put-call parity to be discussed subsequently, the call premium is:

\[C = P - PV[K] + PV[F] = 90 - 1400e^{-0.04} + 1384 = 128.89.\]

Thus the profit from writing a call is: \(128.89e^{0.04} - (S-1400)_+ = 134.15 - (S-1400)_+.\)

For \(S \leq 1400\), the profit is: 134.

For \(S \geq 1400\), the profit is: 1534 - S.

This is equal to Caleb’s profit.
4.22. The future value of the value of Chip’s stock plus put is: \((100)(118 + 13)e^{0.05} = 13,772\).
Chip’s profit is: \(13,772 - 100S - 100(110 - S)_+\).
Dale’s profit is: \((100)(118)e^{0.05} - 100S = 12,405 - 100S\).
For \(S < 110\), Chip’s profit is: \(13,772 - 100S - 100(110 - S) = 2772\).
Chip’s profit is equal to Dale’s when: \(12,405 - 100S = 2772\). ⇒ \(S = 96.33\).
Chip’s profit is greater than Dale’s when: \(S > 96.33\).
Comment: Chip has written a covered put.
The graph of their profits:

```plaintext
Profit

<table>
<thead>
<tr>
<th>Stock Price</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>Dale</td>
</tr>
<tr>
<td>120</td>
<td>Chip</td>
</tr>
<tr>
<td>140</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td></td>
</tr>
</tbody>
</table>
```

Comment: Chip has written a covered put.
4.23. C. Her cost is: $(100)(42) + (100)(2) = 4400$. The future value is: $4400 e^{(1/4)(0.1)} = 4511$. Her payoff per share is: $S + (40-S)^+$. If the stock price at expiration is 40 or less, then her position is worth: $(100)(40) = 4000$. If the stock price at expiration is more than 40, then her position is worth more than 4000. Thus her smallest possible profit is: $4000 - 4511 = -511$.

Comment: A graph of her profit as a function of the stock price at expiration:

![Graph of profit vs. stock price]

4.24. The put would be at-the-money if its strike price is $80$. The put would be out-of-the-money if its strike price were less than $80$. The put would be in-the-money if its strike price were greater than $80$; if the put could be exercised today, then there would be a positive payoff.

4.25. The future value of the option premium is: $11 e^{(2/3)(6\%)} = 11.45$. Thus her profit is: $(90 - S_{2/3})^+ - 11.45$.

Her maximum profit occurs when $S_{2/3} = 0$; her maximum profit is: $90 - 11.45 = 78.55$. Her minimum profit occurs when $S_{2/3} \geq 90$; her maximum profit is: $0 - 11.45 = -11.45$.

Comment: On a purchased put, the maximum payoff and thus maximum profit occurs when the value of the underlying asset at expiration is zero. The maximum profit is finite. Therefore, the maximum possible loss on a written put is also finite.
4.26. B. The profit on the 60-strike put is: \((60-S)^+ - 8.24 e^{0.12} = (60-S)^+ - 9.29\).

The profit on the 70-strike put is: \((70-S)^+ - 12.85 e^{0.12} = (70-S)^+ - 14.49\).

The profit on the 80-strike call is: \((80-S)^+ - 18.30 e^{0.12} = (80-S)^+ - 20.63\).

For \(S \geq 80\), all the puts expire worthless, and the condition is not met.
For \(70 \leq S \leq 80\), the profits are: -9.2, -14.49, and (80-S) - 20.63 = 59.37 - S.

At \(S = 70\), the profit of the 80-strike put is: -10.63 which satisfies the condition.
Set: \(-14.49 = 59.37 - S\). \(\Rightarrow\) \(S = 73.86\). Thus we need \(S < 73.86\).

For \(60 \leq S \leq 70\), the profits are: -9.2, (70-S) - 14.49 = 55.51 - S, and (80-S) - 20.63 = 59.37 - S.

At \(S = 60\), the profit of the 80-strike put is: -0.63 which is bigger than the profit on the 60-strike put.
Set: \(-9.29 = 59.37 - S\). \(\Rightarrow\) \(S = 68.66\). Thus we need \(S > 68.66\).

Thus the desired range is: \(68.66 < S < 73.86\).

Comment: Similar to MFE Sample Introductory Q.11, which instead involves calls.

Here is a graph of the profits:

4.27. If the stock price at expiration is more than $100, then the put expires worthless.
Thus the absolute value of the maximum loss is the future value of the option premium:
\(8 e^{0.03} = 8.24\).

If the stock price at expiration is 0, then the put pays off 100, and you gain:
\(100 - 8 e^{0.03} = 91.76\).
4.28. The future value of the cost of Sonny’s stock plus put is: \((56 + 9)e^{0.08} = 70.41\).
Sonny’s profit is: \(S + (60 - S) = 70.41\).
Cher’s profit is: \(S - 56e^{0.08} = S - 60.66\).
For \(S \leq 60\), Sonny’s profit is: \(S + (60 - S) - 70.41 = -10.41\).
Sonny’s profit is equal to Cher’s when: \(S - 60.66 = -10.41\). \(\Rightarrow S = 50.25\).
Comment: Sonny has established a floor on his profit.

4.29. If the stock price is 0, then you lose 60 due to the put.
Thus taking into account the future value of the option premium, the absolute value of the maximum loss is: \(60 - 7e^{0.1} = 52.26\).
If the stock price is 60 or more, the put expires worthless, and you gain the future value of the put premium: \(7e^{0.1} = 7.74\).
4.30. Graph of the payoff on a European put with strike price of 60, \((60 - S)_+\):

![Graph of the payoff on a European put with strike price of 60, \((60 - S)_+\).](image)

4.31. E. All of these statements are true.

Comment: Covered Call consists of a long stock and short call.

By put-call parity: \(C - P = PV[F] - PV[K]\). \(\Rightarrow -P = PV[F] - C - PV[K]\).

Covered Put consists of a short stock and short put.

By put-call parity: \(C - P = PV[F] - PV[K]\). \(\Rightarrow -PV[F] - P = -C - PV[K]\).

Floor consists of a long stock and a long put.

By put-call parity: \(C - P = PV[F] - PV[K]\). \(\Rightarrow P + PV[F] = C - PV[K]\).

Cap consists of a short stock and a long call.

By put-call parity: \(C - P = PV[F] - PV[K]\). \(\Rightarrow C - PV[F] = P - PV[K]\).

4.32. The future value of the purchase price of the put is: \(6 \exp[(3/4)(5\%)] = 6.23\).

The profit on a written put is: \(6.23 - (80 - S_{3/4})_+\).
4.33. E. Let $S$ be the price of the index in 18 months.
The put premium has future value of: $(15.80)(1.05^{1.5}) = 17.00$.
Since they are negatives of each other, in order for the profit from the long and short positions to be equal they must both be zero.
$$ (120 - S)_+ - 17.00 = 0. \Rightarrow S = 120 - 17.00 = \boxed{103.00}.$$  
Comment: Similar to MFE Sample Introductory Q.12.

4.34. D. If the stock price goes down, the put will have a positive payoff, so Statement #1 is true. Owning a share and also selling a share short, you will be unaffected by stock price movements, so Statement #2 is true.
If the stock price goes down, the call will have no payoff; you can use the money you got from selling the call to offset some of the losses on the stock. Thus Statement #3 is true.
Comment: Buying a put would be the usual way to hedge the risk of the stock price declining.

4.35. a. The actuary’s family has a claim on the future wages of the actuary. If the actuary dies, the value of that claim falls to zero. The term life policy is like a put option which pays off if the actuary dies over the term of the policy.
b. Events other than death can reduce the actuary’s future wages, for example layoff or disability. Under these events, the term life policy will not pay off.
Comment: The analogy is a little strained.

4.36. B. Let $S$ be the price of the index in six months.
The put premium has future value of: $(74.20)(1.02) = 75.68$.
Since they are negatives of each other, in order for the profit from the long and short positions to be equal they must both be zero.
$$ (1000 - S)_+ - 75.68 = 0. \Rightarrow S = 1000 - 75.68 = \boxed{924.32}.$$  

4.37. D. Buying a call in conjunction with a short position is a form of insurance called a cap.
The future value of the cost of the position is: $(10-50)(1.03^2) = -42.44$.
Thus the profit is: $(75 - 60)_+ - 75 + 42.44 = -17.56$.

4.38. C. Premium is; $(100)(5) = 500$.
Payoff is: $(100) (50 -45) = 500$.
Profit is: $500 - 500 = 0$.
Comment: Since we are not given the time until expiration or the interest rate, we have to ignore the time value of money.
Poorly worded; it is intended that the premium is 5 for a put that covers one unit of the index.
4.39. E. The payoff for the 45-strike call is: \( 12 = (S - 45)_+ \). \( \Rightarrow S = 57 \).
Thus the payoff for the 135-strike put is: \( (135 - 57)_+ = 78 \).

4.40. D. If the stock price is at least 50 at expiration, then the put payoff is zero, and the loss is the option premium of \( 500 \).
Comment: Since they did not give us the risk free rate or the time until expiration, we have to ignore the time until expiration.

4.41. B. Initially the investor receives: \( 8 - 1 = 7 \).
Thus with no interest, the profit is: \( 7 + (70 - S)_+ - (80 - S)_+ \).
For \( S \geq 80 \), the profit is 7.
For \( 70 \leq S \leq 80 \), the profit is: \( 7 - (80-S) = S - 73 \).
For \( S \leq 70 \), the profit is: \( 7 + (70-S) - (80-S) = -3 \).
In order for the profit to be zero, \( S = 73 \).
Comment: Since there is no interest, we make no use of the time until expiration.

4.42. A. (A) A in-the-money. \( \Rightarrow S < 70 \). \( \Rightarrow B \) is in-the-money. \( \Rightarrow (A) \) is true.
(B) A at-the-money. \( \Rightarrow S = 70 \). \( \Rightarrow B \) is in-the-money. \( \Rightarrow (B) \) is false.
(C) A in-the-money. \( \Rightarrow S < 70 \). \( \Rightarrow B \) is in-the-money. \( \Rightarrow (C) \) is false.
(D) A is out-of-the-money. \( \Rightarrow S > 70 \). This leaves open any of three possibilities for B, depending on whether \( S < 80 \), \( S = 80 \), or \( S > 80 \). \( \Rightarrow (D) \) is not true.
(E) Similar to (D), (E) is not true.
Comment: If \( S < 80 \), then B is in-the-money. If \( S = 80 \), then B is at-the-money.
If \( S > 80 \), then B is out-of-the-money.

4.43. E. The future price is equally likely to be 125 or 60.
Expected payoff of the put is: \( (0.5)(130 - 125) + (0.5)(130 - 60) = 37.5 \).
Thus the expected profit is: \( 37.5 - (7)(1.03) = 30.29 \).
Comment: For the stated future possible outcomes, the put premium is way too small.
The put premium should be the present value of the expected payoff using the probabilities in the risk neutral environment. Let \( p^* \) be the probability of a move up in stock price in the risk neutral environment, then: \( (100)(1.03) = p^* 125 + (1-p^*) 60. \Rightarrow p^* = 43/65 = 66\% \).
Thus the put premium should be: \( \frac{(0.66)(130 - 125) + (0.34)(130 - 60)}{1.03} = 26.3 \).

4.44. D. If we could exercise the call today, the payoff would be: \( (80-65)_+ = 15 > 0 \).
Thus the call is in-the-money.
If we could exercise the put today, the payoff would be: \( (65-80)_+ = 0 \).
Since \( S > K \), the put is out-of-the-money.
Comment: At-the-money means that \( S = K \).
Section 5, Named Positions

One can buy various combinations of options. The more common such positions have been given names. In each case, you should know: the definition, the payoff diagram, the profit diagram, and the purpose of each of these positions.

**Bull Spreads:**

A spread consists of only calls or only puts. Some options are bought and others are sold.

**Bull Spread:** The purchase of an option together with the sale of an otherwise identical option with a higher strike price. Can construct a bull spread using either puts or calls. The owner of the Bull Spread hopes that the stock price moves up.

For example, Vanessa buys a 90 strike call and sells an otherwise identical 100 strike call. This is an example of a Call Bull Spread. Vanessa hopes the stock price increases. Her payoff is: \((S-90)_{+} - (S-100)_{+}\). Here is a graph of her payoff at expiration:

![Payoff graph](image)

Let us assume that these are one-year calls, with premiums of 12 and 7. Assume that the continuously compounded interest rate is 4%.

Then her premium to buy the bull spread is: 12 - 7 = 5. The future value is 5 \(e^{0.04}\) = 5.20. Her profit is: \((S-90)_{+} - (S-100)_{+} - 5.20.\)

---

72 See Sections 3.3 and 3.4 of *Derivative Markets* by McDonald.
Here is a graph of Vanessa’s profit on her call Bull Spread:

For instead a forward contract on this stock, with a forward price of 96, the profit is $S - 96$:

By buying either the bull spread or the forward contract, one is speculating that the stock price will go up. Compared to the forward contract, the bull spread has less of a loss when the stock price at expiration is really low, but at the cost of less profit when the stock price at expiration is really high. With the bull spread the possible profit and loss are each limited.
Bear Spreads:

Bear Spread: The sale of an option together with the purchase of an otherwise identical option with a higher strike price. Can construct a bear spread using either puts or calls. The owner of the Bear Spread hopes that the stock price moves down.

If Vanessa bought her 90 strike call from Nathan and sold her 100 strike call to Nathan, then Nathan owns a Call Bear Spread. Nathan hopes the stock price declines. His payoff is: \((S-100)_+ - (S-90)_+\). Here is a graph of his payoff at expiration:\(^{73}\)

His profit is: \(5.20 - (S-90)_+ + (S-100)_+\).\(^{74}\)

---

\(^{73}\) Nathan's payoff is minus Vanessa's.

\(^{74}\) Nathan's profit is minus Vanessa's.
If instead one goes short on a forward contract on this stock, with a forward price of 96, the profit is:
\[-(S - 96) = 96 - S.\]

By buying either the bear spread or shorting a forward contract, one is speculating that the stock price will go down. Compared to shorting the forward contract, the bear spread has less of a loss when the stock price at expiration is really high, but at the cost of less profit when the stock price at expiration is really low. With the bear spread the possible profit and loss are each limited.
Box Spread:

Cody buys a $90 strike European call, sells a $90 strike European put, sells a $100 strike European call, and buys a $100 strike European put. The options are on the same stock and have the same expiration date.

<table>
<thead>
<tr>
<th>Strike</th>
<th>Call</th>
<th>Put</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>Buy</td>
<td>Sell</td>
</tr>
<tr>
<td>100</td>
<td>Sell</td>
<td>Buy</td>
</tr>
</tbody>
</table>

His payoff is: \((S-90)^+ - (90-S)^+ - (S-100)^+ + (100-S)^+\) Here is a graph of Cody’s payoff:

![Graph of Cody’s payoff]

The payoff from a box spread is a constant; payoff = the difference in the strikes.

If one were to just buy a $90 strike European call and sell a $90 strike European put, then regardless of the stock price at expiration, you would end up paying $90 and owning the stock. If one were to just buy a $100 strike European put and sell a $100 strike European call, then regardless of the stock price at expiration, you would end selling a share of stock for 100. Thus in combination, for the box spread you buy a share of stock for $90 and then sell that share of stock for $100, for a payoff of $100 - $90 = $10.

Box Spread: Buy a call and sell a put at one strike price, plus at another (higher) strike price sell a call and buy a put.
Let us assume that Cody’s calls are one-year, with premiums of 12 and 7. His one year puts have premiums of 6.47 and 11.08. Assume that the continuously compounded interest rate is 4%. Then his premium to buy the box spread is: 12 - 7 - 6.47 + 11.08 = 9.61. The future value is: 9.61 e^{0.04} = 10.00. His profit is: 10 - 10.00 = 0.

The profit on a Box Spread is zero. A Box Spread is a complicated way to lend at the risk free rate. In this case, Cody gave the broker $9.61 in order to buy the box spread, and received $10 one year later.

Writing a Box Spread is a complicated way to borrow money at the risk free rate. This can be useful to dealers in options, who have low transaction costs.

Ratio Spreads:

Anthony buys a $90-strike European call and sells two 100-strike European calls; all of the options are on the same underlying asset and have the same time until expiration. Anthony’s payoff is: \((S - 90)_+ - 2 (S - 100)_+\):

75 These put premiums satisfy put-call parity to be discussed subsequently, for an assumed initial stock price of 92 and assuming no dividends.
His payoff is zero for low stock prices, positive for stock prices between 90 and 110, and negative for stock prices above 110.

Let us assume that Anthony’s calls are one-year, with premiums of 12 and 7. Assume that the continuously compounded interest rate is 4%.

Then his premium to buy the ratio spread is: $12 - (2)(7) = -2.76$. The future value is: $-2 e^{0.04} = -2.08$. Thus Anthony’s profit is: $(S - 90)_+ - 2 (S - 100)_+ + 2.08$.

Ratio Spread: Buying m of an option and selling n of an otherwise identical option at a different strike.\(^{77}\)

Exercise: How could one set up a ratio spread that has zero premium using these same calls.

[Solution: Buy m of the 90-strike calls and sell n of the 100 strike calls.
Premium = m 12 - n 7. Premium is zero if: n/m = 12/7.
So for example, buy 7 of the 90 strike calls and sell 12 of the 100-strike calls.
Alternately, one could sell 7 of 90-strike calls and buy 12 of the 100-strike calls.
Comment: With zero premium, the profit is equal to the payoff.]

\(^{76}\) He gets money in the door for setting up this particular box spread. The premium could be positive or negative.

\(^{77}\) If m = n = 1, then one has either a Bull Spread or a Bear Spread.
Instead, Brianna buys two $90-strike European call and sells one 100-strike European calls; all of the options are on the same underlying asset and have the same time until expiration. Brianna’s payoff is: $2 (S - 90)_+ - (S - 100)_+$:

Her payoff is zero for stock prices below 90, and positive for stock prices above 90. However, the slope of her payoff diagram is less for stock prices above 100.

Her premium to buy the ratio spread is: $(2)(12) - 7 = 17$. The future value is: $17 e^{0.04} = 17.69$. Brianna’s profit is: $2 (S - 90)_+ - (S - 100)_+ - 17.69$: 

Her payoff is zero for stock prices below 90, and positive for stock prices above 90. However, the slope of her payoff diagram is less for stock prices above 100.
Collars:

Hannah buys a 90-strike European put and sells an otherwise similar 100-strike call. Her payoff is: \((90 - S)_+ - (S - 100)_+\):

Her payoff is similar to that of a short forward contract on the stock; however, in between 90 and 100 her payoff is zero. In this case, Hannah has a bought a collar of width: \(100 - 90 = 10\). The payoff is constant within the width of the collar. Often the width will be larger. Usually the put and call are both out of the money.

Collar: Purchase a put and sell a call with a higher strike price.

In a written collar, one instead sells a put and buys a similar call with a higher strike price. Of course the payoff of a written collar is minus that of a purchased collar.

Let us assume that Hannah’s options in her purchased collar are one-year, with the put premium equal to 6 and the call premium equal to 7. Assume that the continuously compounded interest rate is 4%. Then her premium to buy her collar is: \(6 - 7 = -1\). The future value is: \(-1 e^{0.04} = -1.04\).
Thus Hannah’s profit is: $(90 - S)_+ - (S - 100)_+ + 1.04$:

Hannah’s premium for her collar was small in absolute value. In general, the premium for a collar will have a relatively small absolute value, since selling a call will offset the cost of buying the put. It is possible by choosing appropriate strike prices to construct a collar with no initial cost. As will be discussed in a subsequent section, one can construct a zero-cost collar with any given width.

Let us assume Hannah also owns a share of the stock underlying these options. This is called a collared stock. Then the value of her portfolio in one year is: $S + (90 - S)_+ - (S - 100)_+$:
Her one-year collared stock has protected her against the stock price one year from now being low. This protection is due to her buying the 90-strike put. By selling the 100-strike call, Hannah has reduced her premium, but at the cost of giving up extra profit if the future stock price is above 100.

Assume that the initial stock price is 92 and the stock pays no dividends. Hannah’s options in her purchased collar are one-year, with the put premium equal to 6 and the call premium equal to 7. The continuously compounded interest rate is 4%.

Then her cost to buy her collared stock is: 92 + 6 - 7 = 91. The future value is: $91 e^{0.04} = 94.71$. The value of her portfolio in one year is: $S + (90 - S)_+ - (S - 100)_+$. Her profit on her collared stock is $S + (90 - S)_+ - (S - 100)_+ - 94.71$:

The profit graph of the collared stock is similar to that for a bull spread.
Exercise: Plot the value at expiration of a collared stock with instead 70 and 120 strikes.

[Solution: The value is $S + (70 - S)_+ - (S - 120)_+$.]

![Graph of value at expiration of a collared stock with 70 and 120 strikes.]

**Comment:** Purchase of a share of stock plus a collar with a width of 50.

This graph has the same shape as the payoff on a 70-120 bull spread $(S-70)_+ - (S-120)_+$.

![Graph of payoff on a 70-120 bull spread.]

If one had instead sold a stock short, then one could write a collar in order to get protection. The value of a written collared stock is minus that on a purchased collared stock.

---

78 Buy a 70-strike call and sell a 120-strike call.
Straddles:

Straddle: Purchase a call and the otherwise identical put.

For example, Gene buys a Straddle with $K = 90$. He buys an 90-strike call and a similar 90-strike put. His payoff at expiration is: $(S_T - 90)_+ + (90 - S_T)_+ = |S_T - 90|$. Here is graph of Gene’s payoff:

The further the stock price at expiration is from 90, the larger Gene’s payoff. Gene is hoping there is a large movement in the stock price. In other words, Gene is betting that the stock’s volatility is high.

Let us assume that Gene’s options are one-year, with the put premium equal to 6 and the call premium equal to 12. Assume that the continuously compounded interest rate is 4% Then Gene’s profit is: $(S_T - 90)_+ + (90 - S_T)_+ - (12 + 6) e^{0.04} = |S_T - 90| - 18.73.$

---

In contrast, the seller of a straddle is betting that the stock’s volatility is low.
Here is a graph of Gene’s profit as a function of the stock price at expiration:

The purchaser of a straddle makes money if the stock price moves a lot in either direction, and otherwise loses money.\(^\text{a}\) The reverse is true for the writer of a straddle.

**Strangle:**

Alexis buys an 90-strike put and a similar 100-strike call. Her payoff at expiration is \((90 - S)_+ + (S - 100)_+\):

\(^{a}\) The strike price of the straddle is usually not exactly equal to the initial stock price, but it is usually relatively close.
The payoff is similar to that of a straddle, but between the two strikes the payoff is zero.\footnote{As the two strikes approach each other, a strangle approaches a straddle.}

Strangle: The purchase of a put and a higher strike call with the same time until expiration.

Let us assume that her options are one-year, with the put premium equal to 6 and the call premium equal to 7. Assume that the continuously compounded interest rate is 4%.

Then her profit is: 
\[
(90 - S)_+ + (S - 100)_- - (6 + 7) e^{0.04} = (90 - S)_+ + (S - 100)_- - 13.53.
\]

Here is a graph of Alexis’ profit as a function of the stock price at expiration:
Butterfly Spreads:

Allison buys an 80-strike call, sells two 90-strike calls, and buys one 100-strike call. Her payoff is: 
\[ (S - 80)^+ - 2(S - 90)^+ + (S - 100)^+. \]

For \( S < 80 \), her payoff is 0. For \( 80 \leq S < 90 \), her payoff is \( S - 80 \).
For \( 90 \leq S < 100 \), her payoff is: 
\[ (S-80) - (2)(S-90) = 100 - S. \]
For \( S \geq 100 \), her payoff is: 
\[ (S-80) - (2)(S-90) + (S-100) = 0. \]

The shape of the payoff diagram suggests a butterfly with its wings spread out to either side. Her payoff is positive for small movements of the stock price and zero for large movements. The purchaser of a butterfly spread is speculating that the volatility of the stock will be small.

Butterfly Spread: Buying a \( K \) strike option, selling two \( K + \Delta K \) strike options, and buying a \( K + 2\Delta K \) strike option. The Butterfly Spread can be made up of either similar puts or similar calls.

Let us assume that Allison's calls are one-year, with premiums of 20, 12, and 7.\(^2\)
Then Allison's premium is: 
\[ 20 - (2)(12) + 7 = 3. \]
Assume that the continuously compounded interest rate is 4%.
Then Allison's profit is:
\[ (S - 80)^+ - 2(S - 90)^+ + (S - 100)^+ - 3 e^{0.04} = (S - 80)^+ - 2(S - 90)^+ + (S - 100)^+ - 3.12. \]

\(^2\) The 80-strike call costs the most, while the 100-strike call costs the least.
Here is a graph of Allison’s profit as a function of the stock price at expiration:

The butterfly spread is a limited profit, limited risk strategy. This contrasts with a written straddle, where if the stock price moves a lot in either direction, the loss can be very large.\(^8\)

\(^8\) In a written straddle one sells a put and a call with the same strike; in the graph this strike is 90.
The butterfly spread is a combination of a bull spread with strikes $K$ and $K+\Delta K$, and a bear spread with strikes $K+\Delta K$ and $K+2\Delta K$. For example, Allison purchased a 80-90 call bull spread (buy an 80-strike call and sell a 90-strike call), and also purchased a 90-100 call bear strike (sell a 90-strike call and buy a 100-strike call.)

Exercise: Timothy sells a 90 straddle (sell a 90-strike call and sell a 90-strike put), and also purchases a 80-100 strangle (buys a 80-strike put and buys a 100-strike call.) Compare the payoff to Allison’s 80-90-100 butterfly spread. [Solution: Timothy’s payoff is: $-(S - 90)_+ - (90 - S)_+ + (80 - S)_+ + (S - 100)_+$.]

For $S < 80$, his payoff is: $-(90 - S) + (80 - S) = -10$.
For $80 \leq S < 90$, his payoff is: $-(90 - S) = S - 90$.
For $90 \leq S < 100$, his payoff is: $-(S - 90) = 90 - S$.
For $S \geq 100$, his payoff is: $-(S - 90) + (S-100) = -10$.

These are all exactly 10 less than the payoff from Allison’s butterfly spread.

Comment: Since their payoffs differ by a constant 10, their premiums differ by the discounted value of 10. Thus the profit diagrams for Timothy and Allison are the same.

A butterfly spread has the same profit as a combination of a written straddle with strike $K+\Delta K$, and a purchased strangle with strikes $K$ and $K+2\Delta K$.

One could construct the butterfly spread using puts rather than calls. For example, Samuel buys an 80-strike puts, sells two-90 strike puts, and buys one 100-strike put. His payoff is: $(80 - S)_+ - 2 (90 - S)_+ + (100 - S)_+$.

For $S < 80$, Samuel’s payoff is: $(80 - S) - 2 (90 - S) + (100 - S) = 0$.
For $80 \leq S < 90$, his payoff is: $0 - 2 (90 - S) + (100 - S) = S - 80$.
For $90 \leq S < 100$, his payoff is: $100 - S$.
For $S \geq 100$, his payoff is 0.

This is the same set of payoffs as for Allison’s butterfly spread constructed using calls. Since they result in the same payoffs, the two butterfly spread must have the same premium.\(^84\) Therefore, they must have the same profit.

In general, two otherwise similar butterfly spreads constructed with puts or calls have the same payoff and profit diagrams.

\(^84\) As will be discussed, two portfolios that result in the exact same cashflows at the exact same time must have the same initial cost.
Asymmetric Butterfly Spreads:

Austin buys two 80-strike calls, sells three 90-strike calls, and buys one 110-strike calls. Then Austin’s payoff is: $2(S-80)^+ + (S-110)^+ - 3(S-90)^+$:

![Payoff Graph](image)

This payoff graph is similar to that of a butterfly spread, however it is not symmetric. In an asymmetric butterfly spread, the distances between the strikes are not equal, and therefore the payoff graph slants either towards the left or right.

Asymmetric Butterfly Spread: A butterfly spread in which the the distance between the strike prices is not equal. Let the strike be $K_1$, $K_2$, and $K_3$. Let $\lambda = (K_3 - K_2) / (K_3 - K_1)$.

Then buy $\lambda$ of the low strike, buy $1 - \lambda$ of the high strike, and sell one of the medium strike option.

In this example, $K_1 = 80$, $K_2 = 90$, and $K_3 = 110$. Thus $\lambda = (110-90)/(110-80) = 2/3$. Thus we buy $2/3$ of the low strike, $1/3$ of the high strike, and sell one of the medium strike options.

Austin has bought three 80-90-110 Asymmetric Butterfly Spreads.

Let us assume that Austin’s calls are one-year, with premiums of 20, 12, and 4.

Then Allison’s premium is: $(2)(20) - (3)(12) + 4 = 8$.

---

85 A (symmetric) Butterfly Spread is a special case in which the distance between the strike prices is equal. As with (symmetric) Butterfly Spreads, the options used can be either all puts or all calls.

86 90 is two thirds of the way from 110 to 80.

87 While on the exam and in the textbook they sometimes have non-integer number of options, I prefer to think of integer numbers of options.

88 The 80-strike call costs the most, while the 110-strike call costs the least.
Assume that the continuously compounded interest rate is 4%. Then Austin’s profit is:

\[ 2 (S - 80)_+ + (S - 110)_+ - 3 (S - 90)_+ - 8 e^{0.04} = 2 (S - 80)_+ + (S - 110)_+ - 3 (S - 90)_+ - 8.33. \]

Here is a graph of Austin’s profit as a function of the stock price at expiration:
**Definitions of Named Positions:**

**Bear Spread:** The sale of an option together with the purchase of an otherwise identical option with a higher strike price. Can construct a bear spread using either puts or calls. The owner of the Bear Spread hopes that the stock price moves down.

**Box Spread:** Buy a call and sell a put at one strike price, plus at another (higher) strike price sell a call and buy a put.\(^9\) Equivalent to a zero-coupon bond.\(^9\)

**Bull Spread:** The purchase of an option together with the sale of an otherwise identical option with a higher strike price. Can construct a bull spread using either puts or calls. The owner of the Bull Spread hopes that the stock price moves up.

**Butterfly Spread:** Buying a K strike option, selling two K + ΔK strike options, and buying a K + 2ΔK strike option. Speculating that the stock volatility is low.\(^9\)

**Asymmetric Butterfly Spread:** A butterfly spread in which the distance between the strike prices is not equal. Let the strike be K₁, K₂, and K₃. Let \(\lambda = (K_3 - K_2)/(K_3 - K_1)\). Then buy \(\lambda\) of the low strike, buy 1 - \(\lambda\) of the high strike, and sell 1 of the medium strike.

**Collar:** Purchase a put and sell a call with a higher strike price. Profit is constant for a range of stock prices in the middle. There are an infinite number of zero-cost collars.

**Collared Stock:** Own a share of stock and a collar on that stock. Insurance against the price of the stock going down.\(^9\)

**Ratio Spread:** Buying \(m\) of an option and selling \(n\) of an otherwise identical option at a different strike. Different shapes of profit graphs depending on the ratio of \(m\) and \(n\).

**Straddle:** Purchase a call and the otherwise identical put. Speculating that the stock volatility is high.\(^9\) Larger premium than a strangle.

**Strangle:** Purchase of a put and a higher strike call with the same time until expiration. Speculating that the stock volatility is high. Smaller absolute values of losses and profits than a straddle.

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89 A combination of synthetic long and short forwards. A bull plus bear spread.
90 The profit on a Box Spread is zero. A Box Spread is a complicated way to lend at the risk free rate.
91 Written straddle, speculating that the volatility will be low.
92 A written collared stock involves short selling a share of stock, and protects against the price of the stock increasing.
93 Written butterfly spread, speculating that the volatility will be high.
Examples of Profit Graphs:\textsuperscript{94}

\textbf{Bear Spread}

\begin{center}
\begin{tikzpicture}
  \draw[->] (0,0) -- (4,0) node[below] {Stock Price};
  \draw[->] (0,-4) -- (0,4) node[left] {Profit};
  \draw (0,4) -- (2,0) -- (4,-4);
\end{tikzpicture}
\end{center}

\textbf{Box Spread}

\begin{center}
\begin{tikzpicture}
  \draw[->] (0,0) -- (4,0) node[below] {Stock Price};
  \draw[->] (0,-4) -- (0,4) node[left] {Profit};
  \draw (0,0) -- (4,0);
\end{tikzpicture}
\end{center}

\textbf{Bull Spread}

\begin{center}
\begin{tikzpicture}
  \draw[->] (0,0) -- (4,0) node[below] {Stock Price};
  \draw[->] (0,-4) -- (0,4) node[left] {Profit};
  \draw (0,0) -- (2,4) -- (4,-4);
\end{tikzpicture}
\end{center}

\textsuperscript{94} Written positions have negative the profit of the corresponding purchased position.
Butterfly Spread

Asymmetric Butterfly Spread

Collar
Collared Stock

Straddle

Strangle
Problems:

5.1 (2 points) Graph the payoff on a European call option with a strike price of 100 plus the corresponding put, as a function of the future stock price. Also, what is this position called?

5.2 (3 points) Christopher buys a $60 strike European call, sells two $70 strike European calls, and buys an $80 strike European call. The options are on the same stock and have the same expiration date. Graph the payoff on this portfolio as a function of the future price of the stock. Also, what is this position called?

5.3 (2 points) Victoria buys three $80-strike calls and sells two $60-strike calls. The calls are each European, 6-month, and are on the same stock. The premium for each 60-strike call is 9. The premium for each 80-strike call is 2. The risk-free rate, compounded continuously, is 5%. Graph Victoria’s profit as a function of the stock price at expiration.

5.4 (2 points) Jason buys a $100 strike European put, and sells a $120 strike European put. The puts are on the same stock and have the same expiration date. Graph the payoff on this portfolio as a function of the future price of the stock. Also, what is this position called?
5.5 (3 points) Use the following information:

- The risk-free rate, compounded continuously, is 7%.
- European options on one share of ABC stock expiring in one year have the following prices:

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call option price</th>
<th>Put option price</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>25.92</td>
<td>5.31</td>
</tr>
<tr>
<td>120</td>
<td>15.35</td>
<td>13.38</td>
</tr>
<tr>
<td>140</td>
<td>8.58</td>
<td>25.25</td>
</tr>
</tbody>
</table>

- Jay buys a $120 strike one-year straddle on ABC stock.
- Silent Bob buys a $100-$140 strike one year strangle on ABC stock.

Determine for what stock prices at expiration Jay’s profit exceeds Silent Bob’s.

5.6 (3 points) Melissa buys a $90 strike European call, sells a $90 strike European put, sells a $130 strike European call, and buys a $130 strike European put.

The options are on the same stock and have the same expiration date.

Graph the payoff on this portfolio as a function of the future price of the stock.

Also, what is this position called?

5.7 (2 points) Alexander buys four $70-strike puts and sells three $60-strike puts.

The puts are each European, one year, and are on the same stock.

The premium for each 70-strike put is 9.

The premium for each 60-strike put is 4.

The risk-free rate, compounded continuously, is 6%.

Graph Alexander’s profit as a function of the stock price at expiration.

5.8 (3 points) Amanda buys two $100 strike European call, sells three $110 strike European calls, and buys a $130 strike European call.

The options are on the same stock and have the same expiration date.

Graph the payoff on this portfolio as a function of the future price of the stock.

Also, what is this position called?
5.9 (3 points) Ryan buys one hundred 8-month $70-strike calls on a stock. The premium per call is $17. Ryan writes one hundred 8-month $90-strike calls on the same stock. The premium per call is $10. The risk free continuously compounded annual interest rate is 6%. 
(a) Determine Ryan’s profit if the spot price of the stock at expiration is 60.
(b) Determine Ryan’s profit if the spot price of the stock at expiration is 80.
(c) Determine Ryan’s profit if the spot price of the stock at expiration is 100.

5.10 (2 points) Tiffany buys a $90 strike European call and sells a $90 strike European put. The options are on the same stock and have the same expiration date. Graph the payoff on this portfolio as a function of the future price of the stock.
5.11 (3 points) You are given the following information:

• The current price to buy one share of ABC stock is 60
• The stock does not pay dividends
• European options on one share of ABC stock expiring in two years have the following prices:

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call option price</th>
<th>Put option price</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>28.76</td>
<td>1.51</td>
</tr>
<tr>
<td>60</td>
<td>17.77</td>
<td>6.89</td>
</tr>
<tr>
<td>80</td>
<td>10.72</td>
<td>16.21</td>
</tr>
</tbody>
</table>

A butterfly spread on this stock has the following profit diagram.

The continuously compounded annual risk-free rate is 10%

Determine which of the following will NOT produce this profit diagram?

A. Buy a 40-60 put bull spread, and buy a 60-80 call bear spread.
B. Buy a 40-60 call bear spread, and buy a 60-80 put bull spread.
C. Buy one share of the stock, buy a 40-strike put, buy a 80-strike call, sell two 60-strike calls.
D. Buy a 40-strike put, buy a 80-strike put, sell two 60-strike puts.
E. Buy a 40-strike call, buy a 80-strike call, sell two 60-strike calls.

5.12 (3 points) Heather buys a $70 strike European put and sells a $90 strike European call. The options are on the same stock and have the same expiration date.

Graph the payoff on this portfolio as a function of the future price of the stock.
Also, what is this position called?

5.13 (1 point) Can a Butterfly spread have zero premium?
5.14 (2 points) You are given the following information:
• The current price to buy one share of ABC stock is $86.
• The stock does not pay dividends.
• The risk-free rate, compounded continuously, is 6%.
• European options on one share of ABC stock expiring in six months have the following prices:

<table>
<thead>
<tr>
<th>Option</th>
<th>Strike Price</th>
<th>Put option price</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>80</td>
<td>4.55</td>
</tr>
<tr>
<td>B</td>
<td>90</td>
<td>9.22</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>15.58</td>
</tr>
</tbody>
</table>

Samantha buys one of Option A, sells three of Option B, and buys two of Option C. If the stock price at expiration is 84, what is Samantha’s profit?

5.15 (1 point) ABC stock costs $78.
ABC stock does not pay dividends.
Harold enters into a long position on a $80-strike two-year European call on ABC, and enters into a short position on a $80-strike two-year European put on ABC. Harold pays a net of $6.33 for entering these positions.
What is the continuously compounded risk free rate?
A. 5.0%  B. 5.5%  C. 6.0%  D. 6.5%  E. 7.0%

5.16 (1 point) Can a Bull spread have zero premium?

5.17 (3 points) Allen buys a $70 strike European put, sells four $100 strike European puts, and buys three $110 strike European puts. The options are on the same stock and have the same expiration date. Graph the payoff on this portfolio as a function of the future price of the stock. Also, what is this position called?

5.18 (2 points) Nicholas buys a share of stock, sells a $110 strike European call on that stock, and buys a $110 strike European put on that stock. The options have the same expiration date. Graph the value of this portfolio when the options expire as a function of the future price of the stock.

5.19 (3 points) Kyle owns a share of stock and buys a $80 strike 1-year European put and sells a $100 strike 1-year European call, both on this share of stock. Graph the value on this portfolio as a function of the future price of the stock in one year. Also, what is this position called?

5.20 (2 points) Kevin writes (sells) a 60 strike call and a 60 strike put. The options have the same expiration date. Graph the value of this portfolio when the options expire as a function of the future price of the stock. Also, what is this position called?
5.21 (2 points) Nicole buys one hundred $45-strike puts on a stock.
Nicole writes one hundred $50-strike puts on the same stock.
At expiration, the payoff on Nicole’s position is -210.
Determine the stock price at expiration.

5.22 (2 points) Aaron owns a share of stock of the Charming Prints Company.
The current price of Charming Prints Company stock is $100.
Briefly discuss why might Aaron buy a Collar.

5.23 (2 points) Alexander buys four $70-strike puts and sells three $60-strike puts.
The puts are each European, one year, and are on the same stock.
The premium for each 70-strike put is 9.
The premium for each 60-strike put is 4.
The risk-free rate, compounded continuously, is 6%.
Graph Alexander’s profit as a function of the stock price at expiration.

5.24 (2 points) Lauren buys a 70 strike put and a 90 strike call.
The options have the same expiration date.
Graph the value of this portfolio when the options expire as a function of the future price of the stock.
Also, what is this position called?

5.25 (3 points) You wish to create a box spread using options with strikes of 60 and 80.
Discuss how you could construct the spread using combinations of:
(a) Bull and Bear Spreads
(b) Synthetic long and short forwards
5.26 (1 point)
You buy a European call with a strike price of 80 and sell a European put with a strike price of 80. You also sell a European call with a strike price of 100 and buy a European put with a strike price of 100. All of these options are on the same stock and have the same expiration date. Which of the following is a graph of the payoff on this portfolio as a function of the future price of the stock?

E. None of A, B, C, or D.

5.27 (2 points) Tyler buys one hundred one-year $60-strike calls on a stock. Each of these calls costs $7. Tyler writes one hundred one-year $65-strike calls on the same stock. Each of these calls costs $4. The annual continuously compounded annual interest rate is 5%. Tyler’s profit is $100. Determine the stock price at expiration.
5.28 (2 points) Options traders often refer to straddles and butterflies. Here is an example of each. Straddle: Buy a call with strike price of $100 and simultaneously buy a put with strike price of $100. Butterfly spread: Simultaneously buy one call with strike price of $100, sell two calls with strike price of $110, and buy one call with strike price of $120.

Draw position diagrams for the straddle and butterfly, showing the payoffs from the investor's net position. Each strategy is a bet on variability. Explain briefly the nature of each bet.

5.29 (3 points) 18-month European call options have the following premiums:

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>36</td>
</tr>
<tr>
<td>170</td>
<td>27</td>
</tr>
</tbody>
</table>

Use these call options to create a ratio spread with no initial cost. Plot the profit as a function of the stock price at expiration.

5.30 (1 point) You sell a European call with a strike price of 110, and buy a European put with a strike price of 90. The put and call are on the same stock and have the same expiration date. Which of the following is a graph of the payoff on this portfolio as a function of the future price of the stock?

A. None of A, B, C, or D
5.31 (3 points) Sarah buys one hundred 6-month $100-strike calls on a stock. The premium per call is $7. Sarah writes one hundred 6-month $80-strike calls on the same stock. The premium per call is $16. The risk free continuously compounded annual interest rate is 8%.
(a) Determine Sarah’s profit if the spot price of the stock at expiration is 70.
(b) Determine Sarah’s profit if the spot price of the stock at expiration is 90.
(c) Determine Sarah’s profit if the spot price of the stock at expiration is 110.

5.32 (2 points) You believe that the volatility of a stock is lower than indicated by market prices for options on that stock. You have no opinion as to whether the stock price will increase or decrease. Which of the following should you do?
A. Buy a collar
B. Buy a straddle
C. Buy a ratio spread
D. Sell a bull spread
E. Sell a butterfly spread

5.33 (2 points) David buys and sells European call options, all with the expiry and all on the same underlying asset. He buys 100 60-strike calls, buys 100 70-strike calls, and sells 200 65-strike calls. Graph David’s payoff as a function of the value of underlying at expiration.

5.34 (2 points) An investor buys two $40-strike puts and sells one $50-strike put. The puts are each European and are on the same stock. Graph the payoff of this ratio spread.

5.35 (3 points) Kyle buys a $90 strike 9-month European put for $1.21 and sells a $130 strike 9-month European call for $3.16. The options are on the same stock. \( r = 8\% \), continuously compounded Graph the profit on this portfolio as a function of the future price of the stock in 9 months. Also, what is this position called?

5.36 (3 points) Zachary is using otherwise similar European calls with strikes of \( K_1 < K_2 < K_3 \). Zachary buys \( x \) of the low strike, buys \( y \) of the high strike, and sells 1 of the medium strike. Derive what \( x \) and \( y \) have to be in order to have zero payoff for \( S < K_1 \) and for \( S > K_3 \).
5.37 (3 points) Haley purchases a collared stock with strikes of 120 and 150. The stock pays no dividends and has an initial price of 130. The options expire in 6 months. The premium for the 120-strike put is 3.69. The premium for the 150-strike call is 5.57. The risk-free rate, compounded continuously, is 10%. Graph Haley’s profit as a function of the future price of the stock in six months.

5.38 (3 points) Dennis buys one hundred 18-month $80-strike Straddles on a stock. The premium per call is $8.52. The premium per put is $6.89. The risk-free continuously compounded annual interest rate is 5%.
(a) Determine his profit if the spot price of the stock at expiration is 50.
(b) Determine his profit if the spot price of the stock at expiration is 70.
(c) Determine his profit if the spot price of the stock at expiration is 90.
(d) Determine his profit if the spot price of the stock at expiration is 110.

5.39 (4 points) Thelma buys a one-year 50-70 call bear spread on a stock. Louise buys a one-year 50-70 collar on the same stock. The risk-free continuously compounded annual interest rate is 6%. You are given the following premiums on one-year European options on this stock:

<table>
<thead>
<tr>
<th>Strike</th>
<th>Call Premium</th>
<th>Put Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>15.61</td>
<td>3.30</td>
</tr>
<tr>
<td>70</td>
<td>6.53</td>
<td>13.05</td>
</tr>
</tbody>
</table>
Graph the difference of Thelma’s minus Louise’s profit as a function of the stock price at expiration.

5.40 (5 points) Determine which of the following positions have a potential for an unlimited loss.
- Written Asymmetric Butterfly Spread
- Bear Spread
- Box Spread
- Bull Spread
- Butterfly Spread
- Collar
- Collared Stock
- Ratio Spread
- Straddle
- Written Strangle
Use the following information for the next two questions:

- The risk-free rate, compounded continuously, is 7%.
- European options on one share of ABC stock expiring in one year have the following prices:

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call option price</th>
<th>Put option price</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>25.92</td>
<td>5.31</td>
</tr>
<tr>
<td>120</td>
<td>15.35</td>
<td>13.38</td>
</tr>
<tr>
<td>140</td>
<td>8.58</td>
<td>25.25</td>
</tr>
</tbody>
</table>

5.41 (3 points) Cody purchases a butterfly spread on ABC stock. Draw a graph of Cody’s profit as a function of the stock price at expiration.

5.42 (3 points) Anna writes a 120-strike straddle and buys a 100-140 strangle on ABC stock. Draw a graph of Anna’s profit as a function of the stock price at expiration.

5.43 (2 points) You enter into a short position on 6 put options, each with a strike price of 55. Simultaneously, you enter into a long position on 4 put options, each with a strike price of 70. Calculate the largest possible payoff and the smallest possible payoff for the entire option portfolio.

<table>
<thead>
<tr>
<th>Maximum Payoff</th>
<th>Minimum Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) 0</td>
<td>-70</td>
</tr>
<tr>
<td>(B) 15</td>
<td>-55</td>
</tr>
<tr>
<td>(C) 60</td>
<td>-50</td>
</tr>
<tr>
<td>(D) 70</td>
<td>-15</td>
</tr>
<tr>
<td>(E) Unlimited</td>
<td>0</td>
</tr>
</tbody>
</table>

5.44 (1 point) Can a Bear spread have zero premium?

5.45 (3 points) Use the following information:

- The risk-free rate, compounded continuously, is 8%.
- European options on one share of XYZ stock expiring in nine months have the following prices:

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call option price</th>
<th>Put option price</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>21.15</td>
<td>2.23</td>
</tr>
<tr>
<td>70</td>
<td>14.75</td>
<td>5.24</td>
</tr>
<tr>
<td>100</td>
<td>4.11</td>
<td>22.85</td>
</tr>
</tbody>
</table>

Jose purchases 100 asymmetric butterfly spreads on XYZ stock. Draw a graph of Jose’s profit as a function of the stock price at expiration.
5.46 (2 points) Nathaniel’s Infamous has an exclusive contract to supply hot dogs to the organizers of the annual hot dog eating contest. The contract states that the contest organizers will take delivery of 2000 hot dogs in nine months at the market price. It will cost Nathaniel’s Infamous $3500 to provide 2000 hot dogs. The continuously compounded annual risk-free interest rate is 6%. Nathaniel’s Infamous has decided to hedge as follows (both options are nine-month, European): Buy 2000 2-strike put options for a total of $297, and sell 2000 2.5-strike call options for a total of $429. Determine the range of the profit of Nathaniel’s Infamous.

5.47 (2 points) Abigail buys one hundred $70-strike 9-month puts on a stock with total cost $225. Abigail sells one hundred $90-strike 9-month puts on the same stock with total value $1189. The continuously compounded risk free rate is 8%. Determine the maximum and minimum profit for Abigail’s position at expiration.

5.48 (2 points) A Ratio Spread uses strikes of 60 and 80. For a stock price at expiration of 90, the payoff is 70. For a stock price at expiration of 110, the payoff is 90. Determine how to create this Ratio Spread.

5.49 (2 points) Connor buys a $170-strike 18-month call on a stock; the call premium is 22. Connor sells a $150-strike 18-month call on the same stock; the call premium is 35. The continuously compounded annual risk free rate is 4%. Determine the stock price at expiration such that his profit is zero. A. 160 B. 161 C. 162 D. 163 E. 164

5.50 (3 points) Laura purchases an asymmetric butterfly spread with strikes of 50, 80, and 100. Draw a graph of Laura’s payoff as a function of the stock price at expiration.

5.51 (CAS5B, 11/94, Q.28) (2 points)
a. Graph the overall position diagram when an investor simultaneously buys one call with an strike price of $80, sells two calls with strike prices of $90, and buys one call with an strike price of $100. The current price of the stock is $90. Graph the payoff versus the stock price for the total transaction. List both coordinates of all points where the slope of the graph changes. DO NOT include any other graphs or lines in your final answer.
b. Assuming that markets are efficient and that the investor is rational with no superior knowledge, what is the investor's prediction on the price movement of the stock described in (a)? Briefly explain the logic underlying your answer.
5.52 (CAS5B, 11/94, Q.30) (2.5 points) Mr. Clean has hired Mr. Slob to run his hog farm. Mr. Clean has given Mr. Slob the following incentive plan: if, in exactly one year, the price of hogs has risen by more than 10% from their current price of $50 each, Mr. Clean will pay Mr. Slob a $10,000 bonus. What is the best estimate of the cost of this incentive scheme for Mr. Clean given the following values of one-year European call options on hogs for the strike prices?

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call Option Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$48</td>
<td>$5.38</td>
</tr>
<tr>
<td>$50</td>
<td>$4.11</td>
</tr>
<tr>
<td>$52</td>
<td>$3.05</td>
</tr>
<tr>
<td>$54</td>
<td>$2.20</td>
</tr>
<tr>
<td>$56</td>
<td>$1.54</td>
</tr>
<tr>
<td>$58</td>
<td>$1.04</td>
</tr>
</tbody>
</table>

Note: I have rewritten this past exam question in order to match the current syllabus.

5.53 (CAS5B, 11/98, Q.15) (1 point) What combination of stocks, options and borrowing/lending could be represented by the following position diagram?

1. Sell one share of stock short and borrow the present value of $100.
2. Sell one call with strike price of $100 and sell one put with strike price of $100.
3. Sell one share of stock short, sell two puts with strike price of $100, and lend the present value of $100.

A. 1 B. 2 C. 3 D. 2, 3 E. 1, 2, 3
5.54 (CAS5B, 11/99, Q.30) (2 points) ABC Insurance Company has purchased a reinsurance contract from Reliable Reinsurer providing coverage for $10 million in excess of $20 million. In other words, Reliable Reinsurer has agreed to pay up to, but no more than, $10 million beyond the initial $20 million in loss dollars retained by ABC.

a. (1 point) Draw a position diagram showing the payoff to ABC from the reinsurance as a function of the amount of ABC's total loss. Label both axes.

b. (1 point) If we think of ABC's total loss as the "underlying asset," we can model this reinsurance contract as a mixture of simple options. Describe the option position that replicates the payoffs from the reinsurance contract.

5.55 (CAS5B, 11/99, Q.31) (2 points) Norbert Corporation owns a vacant lot with a book value of $50,000. By a stroke of luck, Norbert finds a buyer willing to pay $200,000 for the lot. However, Norbert must also give the buyer a put option to sell the lot back to Norbert for $200,000 at the end of two years. Moreover, Norbert agrees to pay the buyer $40,000 for a call option to repurchase the lot for $200,000 at the end of two years.

a. (1 point) What would likely happen if the lot is worth more than $200,000 at the end of two years? What if it is worth less than $200,000? Why?

b. (1 point) In effect, Norbert has borrowed money from the buyer. What is the effective annual interest rate per year on the loan? Show all work.

5.56 (MFE Sample Introductory Q.3) Happy Jalapenos, LLC has an exclusive contract to supply jalapeno peppers to the organizers of the annual jalapeno eating contest. The contract states that the contest organizers will take delivery of 10,000 jalapenos in one year at the market price. It will cost Happy Jalapenos $1,000 to provide 10,000 jalapenos and today's market price is 0.12 for one jalapeno. The continuously compounded risk-free interest rate is 6%.

Happy Jalapenos has decided to hedge as follows:

Buy 10,000 0.12-strike put options for 84.30 and sell 10,000 0.14-strike call options for 74.80. Both options are one-year, European.

Happy Jalapenos believes the market price in one year will be somewhere between 0.10 and 0.15 per pepper.

Determine which of the following intervals represents the range of possible profit one year from now for Happy Jalapenos.

A. –200 to 100     B. –110 to 190     C. –100 to 200     D. 190 to 390     E. 200 to 400
5.57 (MFE Sample Introductory Q.8) Joe believes that the volatility of a stock is higher than indicated by market prices for options on that stock. He wants to speculate on that belief by buying or selling at-the-money options. Determine which of the following strategies would achieve Joe’s goal.
A. Buy a strangle
B. Buy a straddle
C. Sell a straddle
D. Buy a butterfly spread
E. Sell a butterfly spread

5.58 (MFE Sample Introductory Q.9) You are given the following information:
• The current price to buy one share of ABC stock is 100
• The stock does not pay dividends
• European options on one share of ABC stock expiring in one year have the following prices:

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call option price</th>
<th>Put option price</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>14.63</td>
<td>0.24</td>
</tr>
<tr>
<td>100</td>
<td>6.80</td>
<td>1.93</td>
</tr>
<tr>
<td>110</td>
<td>2.17</td>
<td>6.81</td>
</tr>
</tbody>
</table>

A butterfly spread on this stock has the following profit diagram.

The continuously compounded risk-free rate is 5%
Determine which of the following will NOT produce this profit diagram?
A. Buy a 90 put, buy a 110 put, sell two 100 puts
B. Buy a 90 call, buy a 110 call, sell two 100 calls
C. Buy a 90 put, sell a 100 put, sell a 100 call, buy a 110 call
D. Buy one share of the stock, buy a 90 call, buy a 110 put, sell two 100 puts
E. Buy one share of the stock, buy a 90 put, buy a 110 call, sell two 100 calls.
5.59 (MFE Sample Introductory Q.15) The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%.

You enter into a short position on 3 call options, each with 3 months to maturity, a strike price of 35, and an option premium of 6.13.

Simultaneously, you enter into a long position on 5 call options, each with 3 months to maturity, a strike price of 40, and an option premium of 2.78.

All 8 options are held until maturity.

Calculate the maximum possible profit and the maximum possible loss for the entire option portfolio.

<table>
<thead>
<tr>
<th>Maximum Profit</th>
<th>Maximum Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) 3.42</td>
<td>4.58</td>
</tr>
<tr>
<td>(B) 4.58</td>
<td>10.42</td>
</tr>
<tr>
<td>(C) Unlimited</td>
<td>10.42</td>
</tr>
<tr>
<td>(D) 4.58</td>
<td>Unlimited</td>
</tr>
<tr>
<td>(E) Unlimited</td>
<td>Unlimited</td>
</tr>
</tbody>
</table>

5.60 (MFE Sample Introductory Q.16) The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%.

The following table shows call and put option premiums for three-month European options of various exercise prices:

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>Call Premium</th>
<th>Put Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>6.13</td>
<td>0.44</td>
</tr>
<tr>
<td>40</td>
<td>2.78</td>
<td>1.99</td>
</tr>
<tr>
<td>45</td>
<td>0.97</td>
<td>5.08</td>
</tr>
</tbody>
</table>

A trader interested in speculating on volatility in the stock price is considering two investment strategies.

The first is a 40-strike straddle.

The second is a strangle consisting of a 35-strike put and a 45-strike call.

Determine the range of stock prices in 3 months for which the strangle outperforms the straddle.

(A) The strangle never outperforms the straddle.

(B) $33.56 < S_T < 46.44$

(C) $35.13 < S_T < 44.87$

(D) $36.57 < S_T < 43.43$

(E) The strangle always outperforms the straddle.
5.61 (MFE Sample Introductory Q.17) The current price for a stock index is 1,000. The following premiums exist for various options to buy or sell the stock index six months from now:

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call Premium</th>
<th>Put Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>950</td>
<td>120.41</td>
<td>51.78</td>
</tr>
<tr>
<td>1,000</td>
<td>93.81</td>
<td>74.20</td>
</tr>
<tr>
<td>1,050</td>
<td>71.80</td>
<td>101.21</td>
</tr>
</tbody>
</table>

Strategy I is to buy the 1,050-strike call and to sell the 950-strike call.
Strategy II is to buy the 1,050-strike put and to sell the 950-strike put.
Strategy III is to buy the 950-strike call, sell the 1,000-strike call, sell the 950-strike put, and buy the 1,000-strike put.
Assume that the price of the stock index in 6 months will be between 950 and 1,050.
Determine which, if any, of the three strategies will have greater payoffs in six months for lower prices of the stock index than for relatively higher prices.
(A) None  
(B) I and II only  
(C) I and III only  
(D) II and III only  
(E) The correct answer is not given by (A), (B), (C), or (D)

5.62 (MFE Sample Introductory Q.39) Determine which of the following strategies creates a ratio spread, assuming all options are European.
(A) Buy a one-year call, and sell a three-year call with the same strike price.
(B) Buy a one-year call, and sell a three-year call with a different strike price.
(C) Buy a one-year call, and buy three one-year calls with a different strike price.
(D) Buy a one-year call, and sell three one-year puts with a different strike price.
(E) Buy a one-year call, and sell three one-year calls with a different strike price.

5.63 (MFE Sample Introductory Q.43) You are given:
i) An investor short-sells a non-dividend paying stock that has a current price of 44 per share.
ii) This investor also writes a collar on this stock consisting of a 40-strike European put option and a 50-strike European call option. Both options expire in one year.
iii) The prices of the options on this stock are:

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call option</th>
<th>Put option</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>8.42</td>
<td>2.47</td>
</tr>
<tr>
<td>50</td>
<td>3.86</td>
<td>7.42</td>
</tr>
</tbody>
</table>
iv) The continuously compounded risk-free interest rate is 5%.
v) Assume there are no transaction costs.
Calculate the maximum profit for the overall position at expiration.
(A) 2.61  (B) 3.37  (C) 4.79  (D) 5.21  (E) 7.39
Box spreads are used to guarantee a fixed cash flow in the future. Thus, they are purely a means of borrowing or lending money, and have no stock price risk. Consider a box spread based on two distinct strike prices (K, L) that is used to lend money, so that there is a positive cost to this transaction up front, but a guaranteed positive payoff at expiration. Determine which of the following sets of transactions is equivalent to this type of box spread.

(A) A long position in a (K, L) bull spread using calls and a long position in a (K, L) bear spread using puts.

(B) A long position in a (K, L) bull spread using calls and a short position in a (K, L) bear spread using puts.

(C) A long position in a (K, L) bull spread using calls and a long position in a (K, L) bull spread using puts.

(D) A short position in a (K, L) bull spread using calls and a short position in a (K, L) bear spread using puts.

(E) A short position in a (K, L) bull spread using calls and a short position in a (K, L) bull spread using puts.
5.65 (MFE Sample Introductory Q.59) An investor has a long position in a non-dividend-paying stock, and additionally, has a long collar on this stock consisting of a 40-strike put and 50-strike call. Determine which of these graphs represents the payoff diagram for the overall position at the time of expiration of the options.
5.66 (MFE Sample Introductory Q.67) Consider the following investment strategy involving put options on a stock with the same expiration date.

i) Buy one 25-strike put
ii) Sell two 30-strike puts
iii) Buy one 35-strike put

Calculate the payoffs of this strategy assuming stock prices (i.e., at the time the put options expire) of 27 and 37, respectively.

(A) -2 and 2 (B) 0 and 0 (C) 2 and 0 (D) 2 and 2 (E) 14 and 0

5.67 (MFE Sample Introductory Q.74)

Consider an airline company that faces risk concerning the price of jet fuel. Select the hedging strategy that best protects the company against an increase in the price of jet fuel.

(A) Buying calls on jet fuel
(B) Buying collars on jet fuel
(C) Buying puts on jet fuel
(D) Selling puts on jet fuel
(E) Selling calls on jet fuel

5.68 (CAS8, 5/09, Q.21) (2.25 points) Given the following information about a box spread related to 1,000 shares of Company XYZ stock:

- The current price of Company XYZ's stock is $100.
- The strike prices of the European call options underlying the box spread are $110 and $120.
- The time to maturity of the box spread is 1 year.
- The continuously compounded risk-free rate is 5% per annum.

Investor A is willing to purchase the box spread from you for $9,750.
Investor B is willing to sell the box spread to you for $9,750.

Assume there are no taxes or transaction costs and you can borrow or lend at the risk-free rate.

a. (1.5 point) Explain whether you purchase or sell the box spread.
Calculate the profit you earn. Show all work.

b. (0.75 point) Assume the options underlying the box spread are American instead of European. The investor with whom you entered into the box spread transaction in part a. above believes the price of Company XYZ will not decrease.

Explain the investor's expected actions immediately after entering the box spread transaction with you.
5.1. This position is called a straddle. Graph of the payoff of a European call plus put each with strike price of 100, $E[(S - 100)_+] + E[(100 - S)_+]$:

Comment: This straddle pays a large amount if the future stock price differs a lot from $100. If $100 is the current price, this is one way to “speculate on volatility.” Similar to Figure 3.10 in Derivatives Markets by McDonald.
5.2. This position is called a Butterfly Spread.
The payoff for the portfolio is: $(S_T - 60)^+ - 2(S_T - 70)^+ + (S_T - 80)^+.$
If $S_T \leq 60$, then the payoff is nothing.
If $70 \geq S_T > 60$, then the payoff is: $S_T - 60$.
If $80 \geq S_T > 70$, then the payoff is: $(S_T - 60) - 2(S_T - 70) = 80 - S_T$.
If $S_T > 80$, then the payoff is: $(S_T - 60) - 2(S_T - 70) + (S_T - 80) = 0$.
A graph of the payoff:

5.3. Her premium is: $(3)(2) - (2)(9) = -12$. The future value is: $-12 e^{(1/2)(0.05)} = -12.30$.
Thus her profit is: $3(S-60)^+ - 2(S-80)^+ + 12.30$.

Comment: This is a ratio spread.
5.4. This position is called a Put Bull Spread.
The payoff for the portfolio is: \((100 - S_T)_+ - (120 - S_T)_+\).
If \(S_T \geq 120\), then the payoff is nothing.
If \(120 > S_T \geq 100\), then the payoff is: \(-(120 - S_T) = S_T - 120\).
If \(100 > S_T\), then the payoff is: \((100 - S_T) - (120 - S_T) = -20\).

A graph of the payoff:

Comment: The premium for the 100 strike put is less than the premium for the 120 strike put.
Joe gained money from setting up this portfolio. Joe is hoping that the future stock price will be at least 120. Similar to Figure 3.7 in Derivatives Markets by McDonald.
5.5. Jay buys a 120-strike put and a 120-strike call.
Jay’s premium is: $15.35 + 13.38 = 28.73$. The future value is: $28.73 e^{0.07} = 30.81$.
Jay’s profit is: $(S-120)_+ + (120 - S)_+ - 30.81 = |S-120| - 30.81$.
Silent Bob buys a 100-strike put and a 140-strike call.
Silent Bob’s premium is: $5.31 + 8.58 = 13.89$. The future value is: $13.89 e^{0.07} = 14.90$.
Silent Bob’s profit is: $(S-140)_+ + (100 - S)_+ - 14.90$.
For $S < 100$, Jay’s profit is: $(120 - S) - 30.81 = 89.19 - S$.
Silent Bob’s profit is: $(100 - S) - 14.90 = 85.10 - S$. So Jay’s profit is more.
For $100 \leq S \leq 120$, Jay’s profit is: $(120 - S) - 30.81 = 89.19 - S$.
Silent Bob’s profit is: $-14.90$.
Set them equal: $-14.90 = 89.19 - S$. $\Rightarrow$ $S = 104.09$. Jay’s profit is more for $S < 104.09$.
For $120 \leq S \leq 140$, Jay’s profit is: $(S - 120) - 30.81 = S - 150.81$.
Silent Bob’s profit is: $-14.90$.
Set them equal: $-14.90 = S - 150.81$. $\Rightarrow$ $S = 135.91$. Jay’s profit is more for $S > 135.91$.
For $S > 140$, Jay’s profit is: $(S - 120) - 30.81 = S - 150.81$.
Silent Bob’s profit is: $(S - 140) - 14.90 = S - 154.90$. So Jay’s profit is more.
In summary, Jay’s profit is more than Silent Bob’s for $S < 104.09$ and for $S > 135.91$.
Comment: A graph of their profits:
5.6. This position is called a Box Spread.
The payoff for the portfolio is: $(S_T - 90)_+ - (90 - S_T)_+ - (S_T - 130)_+ + (130 - S_T)_+$.
If $S_T \leq 90$, then the payoff is: $-(90 - S_T) + (130 - S_T) = 40$.
If $130 \geq S_T > 90$, then the payoff is: $(S_T - 90) + (130 - S_T) = 40$.
If $S_T > 130$, then the payoff is: $(S_T - 90) - (S_T - 130) = 40$. A graph of the payoff:

Comment: The box-spread has a risk free payoff; buying a box-spread is equivalent to buying a bond. Writing a box-spread is equivalent to borrowing money.

5.7. His premium is: $(4)(9) - (3)(4) = 24$. The future value is: $24 e^{(1)(0.06)} = 25.48$.
Thus his profit is: $4 (70-S)_+ - 3 (60-S)_+ - 25.48$.

Comment: This is a ratio spread.
5.8. This position is called an Asymmetric Butterfly Spread.
The payoff for the portfolio is: \(2(S_T - 100)_+ - 3(S_T - 110)_+ + (S_T - 130)_+.\)
If \(S_T \leq 100\), then the payoff is nothing.
If \(110 \geq S_T > 100\), then the payoff is: \(2(S_T - 100)\).
If \(130 \geq S_T > 110\), then the payoff is: \(2(S_T - 100) - 3(S_T - 110) = 130 - S_T\).
If \(S_T > 130\), then the payoff is: \(2(S_T - 100) - 3(S_T - 110) + (S_T - 130) = 0\). A graph of the payoff:

Comment: As will be discussed, such an Asymmetric Butterfly Spread may be used to take advantage of certain arbitrage opportunities. \(\lambda = (130 - 110)/(130 - 100) = 2/3\).
Buy \(\lambda\) of the lowest strike, sell 1 of the middle strike, and buy \((1 - \lambda)\) of the highest strike.
In this case, buy \(2/3\) of the 100 strike, sell 1 of the 110 strike, and buy \(1/3\) of the 130 strike.
Here Amanda has multiplied this position by three.
5.9. The future value of the option premiums is: 
(17-10) \exp[(2/3)(6\%)] = 7.29.

(a) (100) \{(60 - 70)_+ - (60 - 90)_+ - 7.29\} = -\$729.
(b) (100) \{(80 - 70)_+ - (80 - 90)_+ - 7.29\} = \$271.
(c) (100) \{(100 - 70)_+ - (100 - 90)_+ - 7.29\} = \$1271.

Comment: Ryan has purchased 100 call bull spreads.

Here is a graph of Ryan’s profit as a function of the stock price at expiration:

![Graph of Ryan's profit]

5.10. The payoff for the portfolio is: 
(S_T - 90)_+ - (90 - S_T)_+.

If S_T ≤ 90, then the payoff is: -(90 - S_T) = S_T - 90. If S_T > 90, then the payoff is: S_T - 90.

A graph of the payoff:

![Graph of payoff]

Comment: This is the same payoff as on a long forward contract with a forward price of 90. I only graphed from a future stock price of 60 to 120. If S_T = 60, then the person to whom Tiffany sold the put will require Tiffany to buy the stock for 90 from this person, even though the stock is only worth 60. If S_T = 60, then Tiffany has a payoff of 60 - 90 = -30.
5.11. B. Portfolio A: (buy 40-strike put & sell 60-strike put) + (sell 60-strike call & buy 80-strike call).
The cost to set up portfolio A is: 1.51 + 10.72 - 17.77 - 6.89 = -12.43.
If \( S_2 < 40 \), then the profit is: \((40 - S_2) - (60 - S_2) + 12.43e^{0.2} = -4.82\).
If \( 40 \leq S_2 < 60 \), then the profit is: \(-(60 - S_2) + 12.43e^{0.2} = S_2 - 44.82\).
If \( S_2 = 60 \), then the profit is: \(12.43e^{0.2} = 15.18\).
If \( 60 < S_2 \leq 80 \), then the profit is: \(-(S_2 - 60) + 12.43e^{0.2} = 75.18 - S_2\).
If \( 80 < S_2 \), then the profit is: \(-4.82\).

Portfolio B: (sell 40-strike call and buy 60-stripe call) + (buy 60-strike put and sell 80-stripe put).
The cost to set up portfolio B is: 17.77 + 6.89 - 28.76 - 16.21 = -20.31.
If \( S_2 < 40 \), then the profit is: \((60 - S_2) - (80 - S_2) + 20.31e^{0.2} = 4.81\).
If \( 40 \leq S_2 < 60 \), then the profit is: \(-(S_2 - 40) + (60 - S_2) - (80 - S_2) + 20.31e^{0.2} = 44.81 - S_2\).
If \( S_2 = 60 \), then the profit is: \(44.81 - 60 = -15.19\).
If \( 60 < S_2 \leq 80 \), then the profit is: \(-(S_2 - 40) + (S_2 - 60 ) - (80 - S_2) + 20.31e^{0.2} = S_2 - 75.19\).
If \( 80 < S_2 \), then the profit is: \(-4.82\).
Portfolio B does not match the given graph; in fact the profit is negative that in the graph.
The cost to set up portfolio C is: 60 + 1.51 + 10.72 - (2)(17.77) = 36.69.
If \( S_2 < 40 \), then the profit is: \(S_2 + (40 - S_2) - 36.69e^{0.2} = -4.81\).
If \( 40 \leq S_2 < 60 \), then the profit is: \(S_2 - 36.69e^{0.25} = S_2 - 44.81\).
If \( S_2 = 60 \), then the profit is: \(60 - 44.81 = 15.19\).
If \( 60 < S_2 \leq 80 \), then the profit is: \(S_2 - 2(S_2 - 60) - 36.69e^{0.2} = 75.19 - S_2\).
If \( 80 < S_2 \), then the profit is: \(S_2 - 2(S_2 - 60) - 36.69e^{0.2} = 75.19 - S_2\).
The cost to set up portfolio D is: 1.51 + 16.21 - (2)(6.89) = 3.94.
If \( S_2 < 40 \), then the profit is: \((40 - S_2) + (80 - S_2) - 2(60 - S_2) - 3.94e^{0.2} = -4.81\).
If \( 40 \leq S_2 < 60 \), then the profit is: \((80 - S_2) - 2(60 - S_2) - 3.94e^{0.2} = S_2 - 44.81\).
If \( S_2 = 60 \), then the profit is: \(20 - 3.94e^{0.2} = 15.19\).
If \( 60 < S_2 \leq 80 \), then the profit is: \((80 - S_2) - 3.94e^{0.2} = 75.19 - S_2\).
If \( 80 < S_2 \), then the profit is: \(-3.94e^{0.2} = -4.81\).
The cost to set up portfolio E is: \(28.76 + 10.72 - (2)(17.77) = 3.94\).
If \(S_2 < 40\), then the profit is: \(-3.94e^{0.2} = -4.81\).
If \(40 \leq S_2 < 60\), then the profit is: \((S_2 - 40) - 3.94e^{0.2} = S_2 - 44.81\).
If \(S_2 = 60\), then the profit is: \(20 - 3.94e^{0.2} = 15.19\).
If \(60 < S_2 \leq 80\), then the profit is: \((S_2 - 40) - 2(S_2 - 60) - 3.94e^{0.2} = 75.19 - S_2\).
If \(80 < S_2\), then the profit is: \((S_2 - 40) - 2(S_2 - 60) + (S_2 - 80) - 3.94e^{0.2} = -4.81\).

Comment: Similar to MFE Sample Introductory Q.9.

5.12. This position is called a Collar.
The payoff for the portfolio is: \((70 - S_T)_+ - (S_T - 90)_+\).
If \(S_T \leq 70\), then the payoff is: \(70 - S_T\).
If \(90 \geq S_T > 70\), then the payoff is: 0.
If \(S_T > 90\), then the payoff is: \(S_T - 90\).

A graph of the payoff:

Comment: Similar to Figure 3.8 in Derivatives Markets by McDonald.
5.13. Since the payoff from a butterfly spread is zero or positive, its premium must be positive.

Butterfly Spread

![Butterfly Spread Diagram]

Comment: Butterfly Spread consists of buying a K strike option, selling two K + \(\Delta K\) strike options, and buying a K + 2\(\Delta K\) strike option.


The future value is: 8.05 \(e^{(1/2)(0.06)}\) = 8.30.

The payoff is: \((80 - S)_+ - 3 (90 - S)_+ + 2 (100 - S)_+ = (80 - 84)_+ - 3 (90 - 84)_+ + 2 (100 - 84)_+\)

= 0 - (3)(6) + (2)(16) = 14.

Profit is: 14 - 8.30 = $5.70.

Comment: This is an asymmetric butterfly spread. The profit diagram is:

![Profit Diagram]
5.15. B. Harold’s bought a call and sold the otherwise similar put. 
The payoff on Harold’s position is: \((S_2 - 80)_+ - (80 - S_2)_+ = S_2 - 80\).
Since ABC pays no dividends, the prepaid forward price for \(S_2\) is just \(S_0 = 78\).
The prepaid forward price to receive $80 two years from now is \(80e^{-2r}\).
Therefore, the price for Harry’s position is: \(78 - 80e^{-2r}\).
Set \(6.33 = 78 - 80e^{-2r}\). \(\Rightarrow r = 5.5\%\).

5.16. Bull spread: sell an option and buy an otherwise similar option with a higher strike.
For a call bull spread, for \(K_2 > K_1\), the payoff is: \((S - K_2)_+ - (S - K_1)_+ \leq 0\).
Since the payoff from a call bull spread is zero or negative, its premium must be negative.
For a put bull spread, for \(K_2 > K_1\), the payoff is: \((K_2 - S)_+ - (K_1 - S)_+ \geq 0\).
Since the payoff from a put bull spread is zero or positive, its premium must be positive.
Comment: \(C(K_1) > C(K_2)\) for \(K_2 > K_1\). \(P(K_1) < P(K_2)\) for \(K_2 > K_1\).

5.17. This position is called a Asymmetric Butterfly Spread.
The payoff for the portfolio is: \((70 - S_T)_+ - 4(100 - S_T)_+ + 3(110 - S_T)_+\).
If \(S_T \leq 70\), then the payoff is: \((70 - S_T) - 4(100 - S_T) + 3(110 - S_T) = 0\).
If \(100 \geq S_T > 70\), then the payoff is: \(-4(100 - S_T) + 3(110 - S_T) = S_T - 70\).
If \(110 \geq S_T > 100\), then the payoff is: \(3(110 - S_T)\).
If \(S_T > 110\), then the payoff is nothing.
A graph of the payoff:
5.18. The value of this portfolio when the options expire is: \( S_T - (S_T - 110)^+ + (110 - S_T)^+ \).
If \( S_T \leq 110 \), then the value is: \( S_T + (110 - S_T) = 110 \).
If \( S_T > 110 \), then the payoff is: \( S_T - (S_T - 110) = 110 \).
A graph of the value:

5.19. This is called a collared stock.
Thus the value is: \( S + (80-S)^+ - (S - 100)^+ \):

A graph of the value:
5.20. This position is called a written straddle. The payoff is: 
\[-(S - 60)_+ - (60 - S)_+.\]

5.21. The payoff is: 
\[100 (45 - S)_+ - 100 (50 - S)_+.\]

For \( S \geq 50 \), the payoff is zero.
For \( S \leq 45 \), the payoff is: 
\[100(45 - S) - 100(50- S) = -500.\]
For \( 50 > S > 45 \), the payoff is: 
\[-100(50 - S).\]

Set 
\[-210 = -100(50 - S).\]
\[\Rightarrow S = 50 - 2.10 = 47.9.\]

Comment: Nicole has purchased 100 put bull spreads. Here is a graph of her payoff:

While Nicole’s payoff is zero or negative, the premium for the 50-strike puts she sold was more than the premium of the 45-strike puts she bought. Thus initially Nicole got a net positive amount of money to set up her position. For example, if each 50-strike put cost $6 and each 45-strike put costs $4, then initially Nicole got in the door: 
\[(100)(6 - 4) = $200.\]
Her profit would be the future value of this $200 plus the payoff. Thus for example for a stock price of 49 she would make money.
5.22. Collar: Purchase a put and sell a call with a higher strike price. For example, Aaron might buy a 90 strike put and sell a 110 strike call, each of which expire 6 months from now. (There are many other possible Collars.) Then the value of his portfolio 6 months from would be: $S_5 + (90 - S_5)_+ - (S_5 - 110)_+$. If $S_5 \leq 90$, then Aaron’s portfolio is worth 90. (Aaron will use his put to sell the stock for 90.) If $90 < S_5 < 110$, then Aaron’s portfolio is $S_5$. If $S_5 \geq 110$, then Aaron’s portfolio is worth 110. (The person to whom Aaron sold the call, will use the call to buy the stock for 110 from Aaron.)

![Graph of Stock + Collar Value](image)

By buying this collar, Aaron has limited the value of his position in 6 months to be between 90 and 110. Aaron can not make a lot, but also he can not lose a lot.

Comment: Using the Black-Scholes formula, to be discussed subsequently, if the stock pays no dividends, the stock has a volatility of 30%, and $r = 5\%$, then the premium for this collar is -2.60; in other words, Aaron will make more money from selling the call than he spends buying the put. If instead for example, Aaron had bought a 120 strike put and sold a 140 strike call, each of which expire 2 years from now, then he would have limited the value of his position in 2 years to be between 120 and 140.
5.23. His premium is: \((4)(9) - (3)(4) = 24\). The future value is: \(24 e^{(1)(0.06)} = 25.48\). Thus his profit is: \(4 (70-S)_+ - 3 (60-S)_+ - 25.48\).

![Graph showing profit vs. stock price]

Comment: This is a ratio spread.

5.24. This position is called a strangle. The payoff is: \((70 - S)_+ + (S - 90)_+\).

![Graph showing payoff vs. stock price]

5.25. Buy a call and sell a put at 60-strike, plus at 80-strike sell a call and buy a put.
(a) We could buy a 60-80 call Bull Spread (buy a 60-strike call and sell an 80-strike call), and buy a 60-80 put Bear Spread (sell a 60-strike put and buy an 80-strike put).
(b) We could buy a synthetic forward by buying a 60-strike call and selling a 60 strike put. We could also create a synthetic short forward by selling a 80-strike call and buying an 80-strike put. Together these are equivalent to the box spread.
5.26. B. \((S - 80)^+ - (80 - S)^+ = S - 80.\)
\((S - 100)^+ - (100 - S)^+ = S - 100.\)
\((S - 80)^+ - (80 - S)^+ - \{(S - 100)^+ - (100 - S)^+\} = S - 80 - (S - 100) = 20.\)
Comment: This position is called a Box Spread.

5.27. Initially, Tyler pays \((100)(\$7) = \$700\) for the 60-strike calls, and receives \((100)(\$4) = \$400\) by selling the 65-strike calls. Thus he pays a net of: \(\$700 - \$400 = \$300.\)
The future value is: \(300 e^{0.05} = 315.38.\)
Thus the payoff must be: \(100 + 315.38 = 415.38.\)
The payoff is: \(100 (S - 60)^+ - 100 (S - 65)^+.\)
For \(S \leq 60,\) the payoff is zero.
For \(S \geq 65,\) the payoff is: \(100(S - 60) - 100(S - 65) = 500.\)
For \(65 > S > 60,\) the payoff is: \(100(S - 60).\)
Set \(415.38 = 100(S - 60). \Rightarrow S = 60 + 4.15 = 64.15.\)
Comment: Tyler has purchased 100 call bull spreads. Here is a graph of his profit:

![Graph of profit vs. stock price]
5.28. Payoff diagram for the straddle:

The purchaser of the straddle hopes that the stock price moves a lot; the purchaser is betting that there will be high volatility in the stock price.

Payoff diagram for the butterfly spread:

The purchaser of the butterfly spread hopes that the stock price does not move a lot; the purchaser is betting that there will be low volatility in the stock price.
5.29. We buy $m$ 150-strike calls and sell $n$ 170-strike calls. 

$$0 = m \cdot 36 - n \cdot 27 \Rightarrow \frac{n}{m} = \frac{36}{27} = \frac{4}{3}.$$ 

So for example, we could buy 3 150-strike calls and sell 4 170-strike calls. (Or some multiple.) Since the premium is zero, the profit is equal to the payoff, $3 \cdot (S - 150)^+ - 4 \cdot (S - 170)^+$:

![Graph showing profit vs stock price](image1)

Alternately, we could sell 3 150-strike calls and buy 4 170-strike calls. (Or some multiple.) Then the payoff is minus what it was in the prior case, $4 \cdot (S - 170)^+ - 3 \cdot (S - 150)^+$:

![Graph showing profit vs stock price](image2)

5.30. D. $(90 - S)^+ - (110 - S)^+$ is equal to:

- $110 - S$, for $S > 100$,
- $0$, for $90 < S < 110$
- $90 - S$, for $S < 90$.

Comment: This position is called a Collar.
5.31. The future value of the option premiums is: \( (7-16) \exp[(1/2)(8\%)] = -9.37 \)
(a) \((100) \{(70 - 100)_+ - (70 - 80)_+ + 9.37\} = \$937\).
(b) \((100) \{(90 - 100)_+ - (90 - 80)_+ + 9.37\} = -\$63\).
(c) \((100) \{(110 - 100)_+ - (110 - 80)_+ + 9.37\} = -\$1063\).

Comment: Sarah has purchased 100 call bear spreads.
Here is a graph of Sarah’s profit as a function of the stock price at expiration:

5.32. E. Straddles, Strangles, and Butterfly Spreads are each ways to speculate on volatility.
Buying any of them would be speculating that the stock’s volatility will be high.
Selling any of them would be speculating that the stock’s volatility will be low.
Comment: Similar to MFE Sample Introductory Q.8.
5.33. David's payoff is: $100 (60 - S)_+ + 100 (70 - S)_+ - 200 (65 - S)_+$.

Comment: This is a (symmetric) butterfly spread. Since the payoff is non-negative, the cost to purchase the position must be positive. As will be discussed subsequently, due to convexity of the call premiums:

$C(70) - C(65) > C(65) - C(60)$.  
$\Rightarrow C(60) + C(70) - 2C(65) > 0. \Rightarrow$ The premium for the butterfly spread is positive.

5.34. The payoff is: $2 (40 - S)_+ - (50 - S)_+$:
5.35. The premium for this collar is: $1.21 - 3.16 = -$1.95. The future value is: $-1.95 e^{0.06} = -$2.07. Thus the profit is: $(90 - S)_+ - (S - 130)_+ + 2.07$.

5.36. For $S < K_1$ all of the calls expire worthless, so the payoff is zero. For $S > K_3$ the payoff is: $x(S-K_1) + y(S-K_3) - (S-K_2) = S(x + y - 1) + K_2 - xK_1 - yK_3$.

In order for this payoff to be zero for all $S > K_3$:

0 = x + y - 1. \[\Rightarrow y = 1 - x.\]

0 = K_2 - xK_1 - yK_3. \[\Rightarrow 0 = K_2 - xK_1 - (1-x)K_3. \Rightarrow x = (K_3 - K_2) / (K_3 - K_1).\]

Comment: This is an asymmetric butterfly spread, with $\lambda = (K_3 - K_2) / (K_3 - K_1)$.

Then buy $\lambda$ of the low strike, buy $1 - \lambda$ of the high strike, and sell one of the medium strike option.

If one uses puts rather than calls, then in order to have the payoff be zero for $S < K_1$:

0 = x(K_1 - S) + y(K_3 - S) - (K_2 - S) = xK_1 + yK_3 - K_2 - S(x + y - 1). \[\Rightarrow 0 = x + y - 1. \Rightarrow y = 1 - x.\]

0 = xK_1 + yK_3 - K_2. \[\Rightarrow 0 = xK_1 + (1-x)K_3 - K_2. \Rightarrow x = (K_3 - K_2) / (K_3 - K_1).\]
5.37. She buys a share of stock, buys a 120-strike put, and sells a 150-strike call.
She initially spent: $130 + 3.69 - 5.57 = 128.12$. The future value is: $128.12 e^{0.05} = 134.69$.
The value of her portfolio at expiration is: $S_T + (120 - S_T)_+ - (S_T - 150)_+$.
Thus Haley’s profit is: $S_T + (120 - S_T)_+ - (S_T - 150)_+ - 134.69$:  

![Graph showing profit versus stock price]
5.38. Buy 100 80-strike calls and 100 80-strike puts.

The premium is: \((100)(8.52 + 6.89) = 1541\). The future value is: \(1541 e^{(1.5)(0.05)} = 1661\).

His payoff is: \(100 \{(S_T - 80)_+ + (80 - S)_+\} = 100 |S_T - 80|\).

Thus his profit is: \(100 |S_T - 80| - 1661\).

(a) \(100 \, |50 - 80| - 1661 = 1339\).
(b) \(100 \, |70 - 80| - 1661 = -661\).
(c) \(100 \, |90 - 80| - 1661 = -661\).
(d) \(100 \, |110 - 80| - 1661 = 1339\).

Comment: A graph of his profit as a function of the stock price at expiration:
5.39. Thelma writes a 50-strike call and buys a 70-strike call. Her premium is: 6.53 - 15.61 = -9.08. The future value is: \(-9.08 e^{0.06} = -9.64\). Thelma’s profit is: \((S-70)_+ - (S-50)_+ + 9.64\):

Louise buys a 50-strike put and sells a 70-strike call. Her premium is: 3.30 - 6.53 = -3.23. The future value is: \(-3.23 e^{0.06} = -3.43\). Louise’s profit is: \((50 - S)_+ - (S - 70)_+ + 3.43\):
The difference in their profits is: \((S\ 70)_+ - (S-50)_+ + 9.64 - (50 - S)_+ + (S-70)_+ - 3.43\) 
\[= 2(S-70)_+ - |S-50| + 6.21.\]

**Difference in Profit**

![Graph showing the difference in profit against stock price](image)

5.40. **Collar:** Purchase a put and sell a call with a higher strike price. Payoff is: \((K_1 - S)_+ - (S - K_2)_+.\) This approaches negative infinity as S approaches infinity. Thus the collar has a potential for an unlimited loss.

For a Collared Stock, the payoff is: \(S + (K_1 - S)_+ - (S - K_2)_+.\) which approaches \(K_2\) as S approaches infinity. Thus a Collared Stock, does not have a potential for an unlimited loss.

**Ratio Spread:** Buying \(m\) of an option and selling \(n\) of an otherwise identical option at a different strike. Using calls, the payoff is: \(m (S-K_1)_+ - n (S-K_2)_+.\)

As S very large, the payoff is: \((m-n)S + n K_2 - m K_1.\)

For \(n > m\), the payoff approaches negative infinity as S approaches infinity. Thus for a **Ratio Spread using calls with \(n > m\), there is a potential for an unlimited loss.**

Using puts, the payoff is: \(m (K_1-S)_+ - n (K_2-S)_+.\)

As S very large, the payoff is 0. For S = 0, the payoff is: \(m K_1 - n K_2.\)

Thus using puts in a ratio spread, there is not a potential for an unlimited loss.

**Strangle:** The purchase of a put and a higher strike call with the same time until expiration. In a written strangle, the payoff is: \(-(K_1 - S)_+ - (S - K_2)_+.\)

This approaches negative infinity as S approaches infinity. Thus a **Written Strangle has a potential for an infinite loss.**

**Comment:** A put has a finite maximum payoff, while a call has no maximum payoff. Thus a written put has a maximum loss, while a written call has no maximum possible loss. Thus in this question a necessary but not sufficient condition is that the position involves selling a call.

A written straddle also has a potential for an unlimited loss.
5.41. Cody buys a 100-strike call, sells two 120-strike calls, and buys a 140-strike call.
His premium is: $25.92 - (2)(15.35) + 8.58 = 3.80$. The future value is: $3.80 e^{0.07} = 4.08$.
Thus his profit is: $(S-100)_+ - 2 (S-120)_+ + (S-140)_+ - 4.08$:

Alternately, Cody buys a 100-strike put, sells two 120-strike puts, and buys a 140-strike put.
His premium is: $5.31 - (2)(13.38) + 25.25 = 3.80$. The future value is: $3.80 e^{0.07} = 4.08$.
Thus his profit is: $(100-S)_+ - 2 (120-S)_+ + (140-S)_+ - 4.08$:

Comment: Using put-call parity, one can show that the premiums for otherwise similar butterfly spreads using puts or calls are the same. In any case, their premiums have to be the same, since their payoffs are the same. In general, two otherwise similar butterfly spreads constructed with puts or calls have the same payoff and profit diagrams.
5.42. Anna sells a 120-strike call, sells a 120-strike put, buys a 100-strike put, and buys a 140-strike call. Her payoff is: 
\[(100 - S)^+ + (S - 140)^+ - (S - 120)^+ - (120 - S)^+\]

Her premium is: \(5.31 + 8.58 - 15.35 - 13.38 = -14.84\). The future value is: \(-14.84 \cdot e^{0.07} = -15.92\). Thus her profit is: 
\[(100 - S)^+ + (S - 140)^+ - (S - 120)^+ - (120 - S)^+ + 15.92:\]

Comment: Anna’s payoff is always exactly 20 less than Cody’s. Since their payoffs differ by a constant 20, their premiums differ by the discounted value of 20. Thus the profit diagrams for Cody and Anna are the same.
5.43. C. The payoff is: 4 \((70 - S)_+\) - 6 \((55 - S)_+\).

For \(S < 55\): Payoff = \((4)(70 - S) - 6(55 - S) = 2S - 50\). At \(S = 0\) the payoff is -50.

For \(55 \leq S < 70\): Payoff = \((4)(70 - S) - 0 = 280 - 4S\). At \(S = 55\) payoff is 60.

For \(S \geq 70\): Payoff = 0.

Thus the maximum payoff is 60 and the minimum payoff is -50.

Comment: Similar to MFE Sample Introductory Q.15.

A graph for this ratio spread of the payoff:

5.44. Bear spread: buy an option and sell an otherwise similar option with a higher strike.

For a call bear spread, for \(K_2 > K_1\), the payoff is: \((S - K_1)_+ - (S - K_2)_+ \geq 0\).

Since the payoff from a call bear spread is zero or positive, its premium must be positive.

For a put bear spread, for \(K_2 > K_1\), the payoff is: \((K_1 - S)_+ - (K_2 - S)_+ \leq 0\).

Since the payoff from a put bear spread is zero or negative, its premium must be negative.

Comment: \(C(K_1) > C(K_2)\) for \(K_2 > K_1\). \(P(K_1) < P(K_2)\) for \(K_2 > K_1\).
5.45. \( \lambda = \frac{100 - 70}{100 - 60} = 0.75 \).

Thus Jose buys 75 60-strike, sells 100 70-strike, and buys 25 100-strike options.

If Jose uses calls, his payoff is: \( 75 (S-60)_+ + 25 (S-100)_+ - 100 (S-70)_+ \).

His premium is: \( (75)(21.15) + (25)(4.11) - (100)(14.75) = 214 \).

The future value is: \( 214 e^{(3/4)(0.08)} = 227 \).

Thus his profit is \( 75 (S-60)_+ + 25 (S-100)_+ - 100 (S-70)_+ - 227 \):

![Graph showing profit vs. stock price]

Alternately, if Jose instead uses puts, his payoff is: \( 75 (60-S)_+ + 25 (100-S)_+ - 100 (70-S)_+ \).

His premium is: \( (75)(2.23) + (25)(22.85) - (100)(5.24) = 214 \).

The future value is: \( 214 e^{(3/4)(0.08)} = 227 \).

Thus his profit is: \( 75 (60-S)_+ + 25 (100-S)_+ - 100 (70 -S)_+ - 227 \),

which produces the same graph as before.

Comment: As with butterfly spreads, for asymmetric butterfly spreads, the profit graphs are the same (subject to rounding) whether one uses puts or calls.
5.46. Initially they gain: $429 - 297 = 132$. The future value is: $132 e^{(0.75)(0.06)} = 138$.

Thus their profit is: $2000S - 3500 + 2000 (2 - S)_+ - 2000 (S - 2.5)_+ + 138$.

For $S \leq 2$, the profit is: $2000S - 3500 + 2000 (2 - S) + 138 = 638$.
For $2 \leq S \leq 2.5$, the profit is: $2000S - 3500 + 138 = 2000S - 3362$.
For $S \geq 2.5$, the profit is: $2000S - 3500 - 2000 (S - 2.5) + 138 = 1638$.

The range of possible profit is **638 to 1638**.

Comment: Similar to MFE Introductory Sample Q.3.

Nathaniel’s Infamous has purchased 2000 Collars.

Here is a graph of their profit:
The future value of her cost is: $(225 - 1189) e^{(0.75)(0.08)} = -$1024.

Thus her profit is: $100 (70 - S)_+ - 100 (90 - S)_+ + 1024$.

For $S \leq 70$, her profit is: $(100) (70 - S) - (100) (90 - S) + 1024 = -976$.

For $70 \leq S \leq 90$, her profit is: $-(100) (90 - S) + 1024 = 100S - 7976$.

For $S \geq 90$, her profit is: $1024$.

Her minimum profit is -976 and her maximum profit is 1024.

Comment: Abigail bought a 70-90 strike put bull spread.

A graph of her profit as a function of the stock price at expiration:
5.48. Ratio Spread: Buying m of an option and selling n of an otherwise identical option at a different strike. If puts were used, then the given payoffs would be zero.
Using calls, the payoff is: \( m (S-60)_+ - n (S-80)_+ \).
\[ 70 = m \times 30 - n \times 10, \text{ and } 90 = m \times 50 - n \times 30. \]
\[ 70 = 20m - 20n. \]
\[ \Rightarrow m = n+1. \]
\[ 70 = 30m - 10(m-1). \]
\[ \Rightarrow m = 3. \]
\[ \Rightarrow n = 2. \]
We buy three 60-strike calls and sell two 80 strike calls.
Comment: The payoff diagram of this Ratio Spread:
Thus his profit is: $(S - 170)_{+} - (S - 150)_{+} + 13 e^{1.5 \cdot 0.04}$.

For $S = 170$, his profit is: $13.80 - 20 = -6.20$.
For $S = 150$, his profit is 13.80.
Thus his profit is zero, for some $S$ between 150 and 170.

$0 = -(S - 150) + 13.80 \Rightarrow S = 163.80$.

Comment: Connor bought a 150-170 strike call bear spread.
A graph of his profit as a function of the stock price at expiration:
5.50. \( \lambda = (100 - 80) / (100 - 50) = 0.4. \)

Thus buy 0.4 50-strike, sell one 80-strike, and buy 0.6 100-strike options.

If she uses calls, her payoff is: \( 0.4 (S-50)_+ + 0.6 (S-100)_+ - (S-80)_+ \).

Comment: If she uses puts her payoff is: \( 0.4 (50-S)_+ + 0.6 (100-S)_+ - (80-S)_+ \),

which turns out to be exactly the same.

If instead one bought 4 50-strike options, sold 10 80-strike options, and bought 6 100-strike options, then the payoff would be ten times as much.

As with butterfly spreads, for asymmetric butterfly spreads, the payoff graphs are the same whether one uses puts or calls.
5.51. a. The payoff diagram:

![Payoff Diagram](image)

b. The investor is betting that the volatility of the stock will be less than the market expects. The investor has a large payoff if the stock price moves a small amount from its initial price, and no payoff if the stock price moves a large amount from its initial price.

Comment: This is a Butterfly spread; there would have been a net cost to setting up this position.

5.52. If the price of hogs in one year is $55 or more, then Mr. Slob gets his bonus. We can approximate such a payoff by buying a 54-strike call and selling a 56-strike call.

The payoff will be:

$$
\begin{cases}
0 & \text{if } S < 54 \\
S - 54 & \text{if } 54 \leq S \leq 56 \\
2 & \text{if } S > 56
\end{cases}
$$

The cost of one such position is: $2.20 - $1.54 = $0.66.

If $S > 56$ the payoff is $2$, so Mr. Clean would need to buy $10,000 / 2 = 5000$ such positions to fund the bonus.

Thus the cost of this incentive scheme is: $(5000)(0.66) = \$3300$.

Comment: The match between the bonus and the position of calls is approximate. There are other combinations of calls that would also approximate the bonus. An exact match would be provided by 55-strike cash-or-nothing calls, an exotic option to be discussed in a subsequent section.

5.53. D. This is the diagram for selling (writing) a straddle, position #2.

If there are no dividends, then by put call parity, the buying a call is equivalent to buying one share of stock, buying one puts, and borrowing the present value of the strike.

Thus, in this case, position #3 would be equivalent to selling a call and selling a put, position #2.

The value of position #1 at future time $T$ is: $-S_T - 100$, not the given graph.
5.54. a. Payoff is: \( \text{Min}[10 \text{ million}, (L - 20 \text{ million})_+] \).

\[
\begin{align*}
\text{Payoff} & \quad \text{Loss} \\
10 & \quad 15 \\
8 & \quad 20 \\
6 & \quad 25 \\
4 & \quad 30 \\
2 & \quad 35 \\
0 & \quad 40
\end{align*}
\]

b. Buy a 20 strike call and sell a 30 strike call.

\[
(S - 20)_+ - (S - 30)_+ = \begin{cases} 
0, & S < 20 \\
S - 20, & 20 < S < 30 \\
10, & S > 30
\end{cases}
\]

Comment: Many insurance and reinsurance arrangements can be thought of in terms of options.

5.55. a. If in two years the lot is worth more than $200,000, then Norbert will use its call to repurchase the lot for $200,000. If in two years the lot is worth less than $200,000, then the put Norbert gave the buyer will be used to sell the lot to Norbert for $200,000.

b. Norbert gets a net of $200,000 - $40,000 = $160,000 today. In either case, two years from now Norbert will reacquire the lot and will pay the buyer $200,000. Norbert has borrowed $160,000 from the buyer and repaid the buyer $200,000 in 2 years. Interest rate for this loan = \((200,000/160,000)^{1/2} - 1 = 11.8\%\).

5.56. D. The accumulated cost of the hedge is: \((84.30 - 74.80)\exp(.06) = 10.09\). Let \(x\) be the market price.

If \(x < 0.12\), the put is in the money and the payoff is: \(10,000(0.12 - x) = 1,200 - 10,000x\).

The sale of the jalapenos has a payoff of: \(10,000x - 1,000\).

The profit is: \(1,200 - 10,000x + 10,000x - 1,000 - 10.09 = 190\).

From 0.12 to 0.14 neither option has a payoff, and the profit is:

\(10,000x - 1,000 - 10.09 = 10,000x - 1,010\). This ranges from 190 to 390.

If \(x > 0.14\), the call is in the money and the payoff is: \(-10,000(x - 0.14) = 1,400 - 10,000x\).

The profit is: \(1,400 - 10,000x + 10,000x - 1,000 - 10.09 = 390\).

The range of possible profit one year from now is: \(190\) to \(390\).
5.57. **B.** Only straddles can consist of at-the-money options. Higher volatility means that it is more likely that the future stock price will be either very high or very low. If you buy an at-the-money put and call, then you will benefit on the call if the future stock is high and benefit from the put if the future stock price is low. The higher the future stock price the more you benefit from the call and the lower the future stock price the more you would benefit from the put. Therefore, buying a straddle is correct for this speculation.

**Comment:** One could speculate on the belief that the volatility of a stock is lower than indicated by market prices for options on that stock by writing (selling) an at-the-money straddle. In a straddle, you would buy a put and a call, both with the same strike price and time until expiration. In a strangle, you would buy a put and a higher strike call, both with the same time until expiration. In a butterfly spread, you buy a call, sell two calls at a higher strike, and buy a fourth call at a still higher strike; the difference between strikes is the same and all of the calls have the same date of expiration. A butterfly spread may also be put together with puts.

5.58. **D.** The cost to set up portfolio A is: $0.24 + 6.81 - (2)(1.93) = 3.19$.

If $S_1 < 90$, then the profit is: $(90 - S_1) + (110 - S_1) - 2(100 - S_1) - 3.19e^{0.05} = -3.35$.

If $90 < S_1 < 100$, then the profit is: $(110 - S_1) - 2(100 - S_1) - 3.19e^{0.05} = S_1 - 93.35$.

If $S_1 = 100$, then the profit is: $10 - 3.19e^{0.05} = 6.65$.

If $100 < S_1 < 110$, then the profit is: $(110 - S_1) - 3.19e^{0.05} = 106.65 - S_1$.

The cost to set up portfolio B is: $14.63 + 2.17 - (2)(6.80) = 3.20$.

If $S_1 < 90$, then the profit is: $-3.20e^{0.05} = -3.36$.

If $90 < S_1 < 100$, then the profit is: $-(100 - S_1) + 6.32e^{0.05} = S_1 - 93.36$.

If $S_1 = 100$, then the profit is: $6.32e^{0.05} = 6.64$.

If $100 < S_1 < 110$, then the profit is: $-(S_1 - 90) + 6.32e^{0.05} = 106.64 - S_1$.

The cost to set up portfolio C is: $0.24 + 2.17 - 6.80 - 1.93 = -6.32$.

If $S_1 < 90$, then the profit is: $(90 - S_1) - (100 - S_1) + 6.32e^{0.05} = -3.36$.

If $90 < S_1 < 100$, then the profit is: $-(100 - S_1) + 6.32e^{0.05} = S_1 - 93.36$.

If $S_1 = 100$, then the profit is: $6.32e^{0.05} = 6.64$.

If $100 < S_1 < 110$, then the profit is: $-(S_1 - 100) + 6.32e^{0.05} = 106.64 - S_1$.

If $110 < S_1$, then the profit is: $-(S_1 - 100) + (S_1 - 110) + 6.32e^{0.05} = -3.36$. 
The cost to set up portfolio D is: $100 + 14.63 + 6.81 - (2)(1.93) = 117.68$.
If $S_1 < 90$, then the profit is: $S_1 + (110 - S_1) - 2(100 - S_1) - 117.68e^{0.05} = 2S_1 - 213.71$,
not matching the given graph.
If $90 < S_1 < 100$, then the profit is: $S_1 + (S_1 - 90) + (110 - S_1) - 2(100 - S_1) - 117.68e^{0.05}$
$= 3S_1 - 303.71$, not matching the given graph.
If $S_1 = 100$, then the profit is: $100 + 10 + 10 - 117.68e^{0.05} = -3.71$, not matching the given graph.
If $100 < S_1 < 110$, then the profit is: $S_1 + (S_1 - 90) + (110 - S_1) - 117.68e^{0.05} = S_1 - 103.71$, not matching the given graph.
If $110 < S_1$, then the profit is: $S_1 + (S_1 - 90) - 117.68e^{0.05} = 2S_1 - 213.71$, not matching the given graph.
The cost to set up portfolio E is: $100 + 0.24 + 2.17 - (2)(6.80) = 88.81$.
If $S_1 < 90$, then the profit is: $S_1 + (90 - S_1) - 88.81e^{0.05} = -3.36$.
If $90 < S_1 < 100$, then the profit is: $S_1 - 88.81e^{0.05} = S_1 - 93.36$.
If $S_1 = 100$, then the profit is: $100 - 93.36 = 6.64$.
If $100 < S_1 < 110$, then the profit is: $S_1 - 2(S_1 - 100) - 88.81e^{0.05} = 106.64 - S_1$.
If $110 < S_1$, then the profit is: $S_1 - 2(S_1 - 100) + (S_1 - 110) - 88.81e^{0.05} = -3.36$.
Comment: See Exercise 3.18 in Derivatives Markets by McDonald.
For each strike price, put-call parity holds. With no dividends, $C - P = 100 - Ke^{-0.05}$.
5.59. C.  The initial cost to establish this position is: $(5)(2.78) - (3)(6.13) = -4.49$.
You receive 4.49 up front, which after three months grows to: $4.49 \exp[(0.25)(0.08)] = 4.58$.
The profit is: $5 \text{ (S-40)}_+ - 3 \text{ (S-35)}_+ + 4.58$.
For $S < 35$, Profit = $0 - 0 + 4.58 = 4.58$.
For $35 \leq S \leq 40$: Profit = $0 - 3(S-35) + 4.58 = 109.58 - 3S$.
Minimum of $-10.42$ is at $S = 40$, and maximum of 4.58 is at $S = 35$.
For $S > 40$, Profit = $5(S-40) - 3(S-35) + 4.58 = 2S - 90.42$.
Minimum of $-10.42$ is at $S = 40$, and maximum is infinity.
Thus **the maximum loss is 10.42 and the maximum profit is unlimited**.

Comment: A graph for this ratio spread of the profit:
5.60. D. The straddle consists of buying a 40-strike call and buying a 40-strike put. This costs: $2.78 + 1.99 = 4.77$. The future value is: $4.77 \exp(0.02) = 4.87$. The straddle has profit: $(S-40)_+ + (40-S)_+ - 4.87$.

For $S < 40$, the straddle has a profit of: $40 - S - 4.87 = 35.13 - S$.
For $S \geq 40$, the straddle has a profit of: $S - 40 - 4.87 = S - 44.87$

The strangle consists of buying a 35-strike put and a 45-strike call. This costs: $0.44 + 0.97 = 1.41$. The future value is: $1.41 \exp(0.02) = 1.44$. The strangle has profit: $(S-45)_+ + (35-S)_+ - 1.44$.

For $S \leq 35$, the strangle has a profit of $35 - S - 1.44 = 33.56 - S$.
For $35 < S < 45$, the strangle has a profit of $-1.44$.
For $S \geq 45$, the strangle has a profit of $S - 45 - 1.44 = S - 46.44$.

Thus for $S < 35$, the strangle underperforms the straddle.
For $S > 45$, the strangle underperforms the straddle.
For $35 < S < 40$, the strangle outperforms the straddle if:
$-1.44 > 35.13 - S \Rightarrow S > 36.57$.
For $40 < S < 45$, the strangle outperforms the straddle if:
$-1.44 > S - 44.87 \Rightarrow S < 43.43$.

The strangle outperforms the straddle for $36.57 < S < 43.43$.

Comment: The strangle has a smaller maximum loss in exchange for giving up some potential gain compared to the straddle. A graph of their profits:

![Graph of straddle and strangle profits](image)
5.61. **B.** Strategy I is a call bear spread, which perform better when the prices of the underlying asset goes down.
Strategy II is a put bear spread, which perform better when the prices of the underlying asset goes down.
Strategy III is a box spread and the payoff is the same 50, no matter the price of the stock index at expiration.
**Comment:** We make no use of given option premiums or the given current price of the stock index.
For the call bear spread, the payoff at 950 is 0, at 1000 is -50, and at 1050 is -100.
For the put bear spread, the payoff at 950 is 100, at 1000 is 50, and at 1050 is 0.
For the box spread, the payoff at 950 is 50, at 1000 is 50, and at 1050 is 50.

5.62. **E.** Ratio Spread: Buying m of an option and selling n of an otherwise identical option at a different strike. E matches this definition; it is an example of a 3:1 ratio spread.
**Comment:** While m = n is a special case of a Ratio Spread, most people would not refer to this as a Ratio Spread.
A is an example of a Calendar Spread. See Section 12.4 in *Derivative Markets* by McDonald.
5.63. C. The written collar consists of a short 40-strike put and a long 50-strike call. Therefore, including shorting the stock, the initial cost of the position is: 

\[-44 - 2.47 + 3.86 = -42.61.\]

The future value is: 

\[-42.61 \cdot e^{0.05} = -44.79.\]

The value of the portfolio at expiration: 

\[-S - (40 - S)_+ + (S - 50)_+.\]

Thus the profit is: 

\[44.79 + (S - 50)_+ - S - (40 - S)_+.\]

For \(S \leq 40\), the profit is: 

\[44.79 - S - (40 - S) = 4.79.\]

For \(40 \leq S \leq 50\), the profit is: 

\[44.79 - S \leq 4.79.\]

For \(S \geq 50\), the profit is: 

\[44.79 + (S - 50) - S = -5.21.\]

Thus, the maximum profit is \(4.79\).

Comment: This is a written collared stock.

Here is a graph of the profit as function of the stock price at expiration.

![Graph of profit vs. stock price](image)

5.64. A. We buy a K-strike call, sell a K-strike put, sell an L-strike call, buy an L-strike put. The payoff in a box spread is the difference in the strike prices; for the payoff to be positive, as required, we must have \(L > K\). (With a positive payoff, the initial cost must also be positive.)

A long position in a \((K, L)\) bull spread using calls: buy a K-strike call and sell an L-strike call.

A long position in a \((K, L)\) bear spread using puts: buy an L-strike put and sell a K-strike put.

Thus portfolio A matches the box spread.

Comment: Bear Spread: The sale of an option together with the purchase of an otherwise identical option with a higher strike price. Can construct a bear spread using either puts or calls.

The owner of the Bear Spread hopes that the stock price moves down.

Box Spread: Buy a call and sell a put at one strike price, plus at another (higher) strike price sell a call and buy a put.

Bull Spread: The purchase of an option together with the sale of an otherwise identical option with a higher strike price. Can construct a bull spread using either puts or calls.

The owner of the Bull Spread hopes that the stock price moves up.
5.65. E. Collar: buy 40-strike put and sell 50 strike call.
Payoff for the collared stock is: $S + (40-S)_+ - (S - 50)_+$.
For $S \leq 40$, payoff is: $S + (40-S) + 0 = 40$.
For $40 \leq S \leq 50$, the payoff is: $S + 0 + 0 = S$.
For $S \geq 50$, the payoff is: $S + 0 - (50-S) = 50$.
This is graph E.
Comment: (B) could be either a bear spread or a written collared stock.
(C) could be a collar.
(D) could be a written collar.

5.66. C. When the stock price is 27, Payoff = $0 - (2)(30-27) + (35-27) = 2$.
When the stock price is 37, Payoff = $0 - (2)(0) + 0 = 0$.
Comment: This is a put butterfly spread. The payoff is: $(25-S)_+ - 2(30-S)_+ + (35-S)_+$.
Here is a graph of the payoff as a function of the stock price at expiration:

![Payoff Graph]

5.67. A. 
**Buying calls** allows a firm to insure against loss of profit as the price of their input increases.
For example, if one buys a call with a $8 per gallon strike, and if the price of fuel increases to for example $9 per gallon, then one can use the call to buy jet fuel at instead $8 per gallon.
5.68. (a) Box Spread: Buy a call and sell a put at one strike price, plus at another (higher) strike price sell a call and buy a put. For European options, the box spread is equivalent to a zero-coupon bond.

Here the owner of the box spread, would buy a $110 strike call, sell a $110 strike put, sell a $120 strike call, and buy a 120 strike put.

The payoff on this box spread is: $(S_T - 110)_{+} - (110 - S_T)_{+} - (S_T - 120)_{+} + (120 - S_T)_{+}$.

- If $S_T < 110$, then the payoff is: $0 - (110 - S_T) + (120 - S_T) = 10$.
- If $120 > S_T > 110$, then the payoff is: $(S_T - 110) + (120 - S_T) = 10$.
- If $S_T > 120$, then the payoff is: $(S_T - 110) - (S_T - 120) = 10$.

The payoff is always 10; there is no stock price risk.

Thus the appropriate premium for one box spread is: $10 \exp[-0.05] = 9.512$.

Therefore, for 1000 box spreads, the appropriate premium is $9512$.

At $9,750$, the box spread is overpriced.

I would sell the box spread to investor A,

earning a profit now of: $9,750 - 9512 = 238$.

Alternately, I sell the box spread to investor A and invest the money at the risk free rate of 5% for one year, and have: $(9750)(e^{0.05}) = 10,250$. I then pay investor A $(1000)(10) = 10,000$.

I make a profit of $10,250 - $10,000 = $250 in one year.

(b) Investor A owns a $120 strike put as part of the box spread.

Since Investor A believes that the price of the stock will not decrease, he expects this put to have a payoff of $120 - $100 = $20 or less.

Therefore, Investor A will want to lock in his $20 payoff, and Investor A should exercise this $120 strike American put right away.

When he does so, I would have to pay him: $(1000)(120) = 120,000$, for shares of stock that are only worth: $(1000)(100) = 100,000$. (Investor A will hold onto his $110 strike call, and hope it will be in the money sometime during the year.)

(You can also exercise the options you own, such as the $110 strike put, but the question does not ask about that. As will be discussed, in general, it can be hard to decide whether it is optimal to exercise an American option early.)

Comment: Investor A owns a $120 strike put and a $110 strike call, which you are responsible for fulfilling if he exercise them. You own a $110 strike put and a $120 strike call, which Investor A is responsible for fulfilling if you exercise them.

If all of the options are European, then at expiration in one year, regardless of the stock price, you will get a net payoff from all four of the options of -$10, while Investor A gets a net payoff of $10.

In part (b), when all of the options are American, they may be exercised at different times. Now the net payoff is uncertain and depends on when each of the options is exercised and the stock prices at those times. The net payoff to Investor A from all four options will depend on both his and your decisions as well as the stock price path over the next year.