MLC Formula Sheet Summary

Survival Distributions

${}_t p_x$ = the probability someone aged 'x' lives 't' years

${}_t q_x$ = the probability someone aged 'x' dies within 't' years

${}_{t|u} q_x$ = the probability that (x) survives 't' years and then dies in the next 'u' years

Symbol	Equals	Equals	Equals	Equals	Equals
${}_t p_x$	$1 - {}_t q_x$	$\frac{s(x+t)}{s(x)}$	$\frac{l_{x+t}}{l_x}$	$e^{-\int_0^t \mu(x+s)ds}$	$e^{-\int_x^{x+t} \mu(s)ds}$
${}_t q_x$	$1 - {}_t p_x$	$\frac{s(x)-s(x+t)}{s(x)}$	$\frac{l_x - l_{x+t}}{l_x}$	$\frac{{}_t d_x}{l_x}$	$\int_0^t {}_s p_x \cdot \mu(x+s)ds$
${}_t d_x$	$l_x - l_{x+t}$				
${}_t p_0$	$s(t)$				
${}_t q_0$	$F(t)$				
${}_t p_x \, {}_u p_{x+t}$	${}_{t+u} p_x$				
${}_{t\|u} q_x$	${}_t p_x - {}_{t+u} p_x$	${}_{t+u} q_x - {}_t q_x$	$\frac{l_{x+t} - l_{x+t+u}}{l_x}$	${}_t p_x \, {}_u q_{x+t}$	$\int_t^{t+u} {}_s p_x \cdot \mu_{x+s} ds$
$\mu(x+t)$	$\frac{-s'(x+t)}{s(x+t)}$	$-\frac{d}{dt}\ln(s(x+t))$	$\frac{\frac{d}{dt}\, {}_t q_x}{{}_t p_x}$	$\frac{-\frac{d}{dt}\, {}_t p_x}{{}_t p_x}$	$\frac{-\frac{d}{dt}(l_{x+t})}{l_{x+t}}$
$f(x)$	$s(x) \cdot \mu(x)$	$p_0 \cdot \mu(x)$			
$s(x)$	${}_x p_0$	$\frac{l_x}{l_0}$	$1 - F(x)$	$e^{-\int_0^x \mu(s)ds}$	

Valid force of mortality

$s(x) \to 0$ as $x \to \infty$

$\therefore \int_0^\infty \mu(t)dt \to \infty$ as $x \to \infty$

$\int_0^\infty {}_s p_x \cdot \mu(x+s)ds = 1$

$\int_0^t {}_s p_x \cdot \mu(x+s)ds = {}_t q_x$

$\int_t^\infty {}_s p_x \cdot \mu(x+s)ds = {}_t p_x$

$\int_t^{t+u} {}_s p_x \cdot \mu(x+s)ds = {}_{t|u} q_x$

Gompertz's Law	Makeham's Law	Weibull Dist.
$\mu(x) = Bc^x$	$\mu(x) = A + Bc^x$	$\mu(x) = kx^n$

Force of Mortality Shortcuts

If $\mu_x'(t) = \mu_x(t) + k$, then ${}_t p_x' = {}_t p_x e^{-kt}$

If $\mu_x'(t) = \hat{\mu}_x(t) + \tilde{\mu}_x(t)$, then ${}_t p_x' = {}_t \hat{p}_x \, {}_t \tilde{p}_x$

If $\mu_x'(t) = k\mu_x(t)$, then ${}_t p_x' = ({}_t p_x)^k$

Mortality Laws

Name	Constant Force of Mortality	DML (equal # deaths/year)	MDML
$\mu(x)$	μ	$\frac{1}{\omega-x}$	$\frac{\alpha}{\omega-x}$
$\mu(x+t)$	μ	$\frac{1}{\omega-x-t}$	$\frac{\alpha}{\omega-x-t}$
${}_tp_x$	$e^{-\mu t}$ $(p_x)^t$	$\frac{\omega-x-t}{\omega-x}$	$\left(\frac{\omega-x-t}{\omega-x}\right)^{\alpha}$
${}_tq_x$		$\frac{t}{\omega-x}$	
${}_{t\|u}q_x$		$\frac{u}{\omega-x}$	
$f_T(t)$	$\mu e^{-\mu t}$	$\frac{1}{\omega-x}$	
$\mathring{e}_x$	$\frac{1}{\mu}$	$\frac{\omega-x}{2}$	$\frac{\omega-x}{\alpha+1}$
e_x	$\frac{e^{-\mu}}{1-e^{-\mu}}$	$\mathring{e}_x-0.5$	
$Var(T(x))$	$\frac{1}{\mu^2}$	$\frac{(\omega-x)^2}{12}$	$\frac{(\omega-x)^2\alpha}{(\alpha+1)^2(\alpha+2)}$
$s(x)$	$e^{-\mu x}$	$\frac{\omega-x}{\omega}$	$\left(\frac{\omega-x}{\omega}\right)^{\alpha}$
l_x	$l_0e^{-\mu x}$	$k(\omega-x)$	$k(\omega-x)^{\alpha}$
$\mathring{e}_{x:\overline{n}\|}$	$\mathring{e}_x(1-{}_np_x)$ $\frac{1-e^{-\mu n}}{\mu}$	$(n)\,{}_np_x+\left(\frac{n}{2}\right){}_nq_x$	Area of a trapezoid between x-axis, ${}_np_x$, x = 0 and x = n. A = avg. of heights x width
$e_{x:\overline{n}\|}$	$e_x(1-{}_np_x)$	$\mathring{e}_{x:\overline{n}\|}-0.5\,{}_nq_x$	
m_x	μ_x		
$m(x)=\text{median}$		Median = Mean	Median = Mean
$a(x)$		0.5	

Moments

$\overset{\circ}{e}_x = E[T(x)]$	$\int_0^\infty t \cdot {}_tp_x \cdot \mu(x+t)dt$	$\int_0^\infty {}_tp_x dt$	$\int_0^\infty \frac{l_{x+t}}{l_x} dt$
$E[T(x)^2]$	$\int_0^\infty t^2 \cdot {}_tp_x \cdot \mu(x+t)dt$	$2\int_0^\infty t \cdot {}_tp_x dt$	
$Var[T(x)]$	$E(T(x)^2) - \left(\overset{\circ}{e}_x\right)^2$		
$\overset{\circ}{e}_{x:\overline{n}\rvert} = E[T(x) \wedge n]$	$E[\min(T(x), n)]$	$\int_0^n t \cdot {}_tp_x \cdot \mu_{x+t} dt + n \cdot {}_np_x$	$\int_0^n {}_tp_x dt$
$E[(T(x) \wedge n)^2]$	$2\int_0^n t \cdot {}_tp_x dt$	2nd Moment	
e_x	$\sum_{k=1}^{\infty} {}_kp_x$		
$e_{x:\overline{n}\rvert}$	$\sum_{k=1}^{n} {}_kp_x$		
$E[K(x)^2]$	$\sum_{k=1}^{\infty} (2k-1)\,{}_kp_x$		
$E[(K(x) \wedge n)^2]$	$\sum_{k=1}^{n} (2k-1)\,{}_kp_x$		

Recursive Formulas for Life Expectancy

$\overset{\circ}{e}_x = \overset{\circ}{e}_{x:\overline{n}\rvert} + {}_np_x \overset{\circ}{e}_{x+n}$

$\overset{\circ}{e}_{x:\overline{m+n}\rvert} = \overset{\circ}{e}_{x:\overline{m}\rvert} + {}_mp_x \overset{\circ}{e}_{x+m:\overline{n}\rvert}$

$e_x = e_{x:\overline{n}\rvert} + {}_np_x e_{x+n}$

$e_x = p_x + p_x e_{x+1}$

Central Death Rate

T_x = total future lifetime after age 'x'

${}_nL_x$ = total future lifetime b/w ages 'x' and 'x+n' by l_x lives

$\overset{\circ}{e}_x$ = complete expectation of life = $\dfrac{\text{total future lifetimes}}{\text{\# of lives}}$

$\overset{\circ}{e}_{x:\overline{n}|}$ = 'n' year temporary complete life expectancy
= avg # of years lived within the next 'n' years

${}_nm_x = \dfrac{\text{\# deaths in next 'n' years}}{\text{\# people alive during next 'n' years}}$ = Central Death Rate

$a(x)$ = fraction of the year lived by those who die during the year

Symbol	Equals	Equals
T_x	$\int_0^\infty l_{x+t}\,dt$	
${}_nL_x$	$\int_0^n l_{x+t}\,dt$	$T_x - T_{x+n}$
$\overset{\circ}{e}_x$	$\dfrac{T_x}{l_x}$	
$\overset{\circ}{e}_{x:\overline{n}\rceil}$	$\dfrac{{}_nL_x}{l_x}$	
${}_nm_x$	$\dfrac{{}_nd_x}{{}_nL_x}$	
$a(x)$	$\dfrac{L_x - l_{x+1}}{d_x}$	

Fractional Ages

Name	Constant Force of Mort	UDD	Balducci
l_{x+t}	$l_x^{(1-t)} \cdot l_{x+1}^{(t)}$ $l_x(p_x)^t$	$l_x(1-t)+l_{x+1}(t)$ $l_x - td_x$	$\frac{1}{l_{x+t}} = \frac{1}{l_x}(1-t)+\frac{1}{l_{x+1}}(t)$
$\mu_x(t)$	$-\ln(p_x)$	$\frac{q_x}{1-tq_x}$	$\frac{q_x}{1-(1-t)q_x}$
${}_tp_x$	$(p_x)^t$ $e^{-\mu t}$	$1-tq_x$	$\frac{p_x}{1-(1-t)q_x}$
${}_tq_x$	$1-(p_x)^t$	tq_x	$\frac{tq_x}{1-(1-t)q_x}$
${}_tq_{x+s}$		$\frac{tq_x}{1-sq_x}$	$\frac{tq_x}{1-(1-s-t)q_x}$
$L(x)$		$l_x - 0.5d_x$ $l_{x+1}+0.5d_x$ $.5(l_x+l_{x+1})$	
$\mathring{e}_x$		$e_x + 0.5$	
m_x		$\frac{q_x}{1-0.5q_x}$	
$Var(T(x))$		$Var(K)+\frac{1}{12}$	
$\mathring{e}_{x:\overline{1}\rceil}$		$p_x + 0.5q_x$	
$\mathring{e}_{x:\overline{n}\rceil}$		$e_{x:\overline{n}\rceil} + 0.5\,{}_nq_x$	
$a(x)$		0.5	
${}_tp_x \cdot \mu_x(t)$		q_x	

Balducci kills off people the fastest
UDD kills off people the slowest
CF is in between Balducci and UDD

Select and Ultimate Mortality

${}_{t}p_{[x]} =$ conditioned on survival to age x and selection process

$q_{[28]} + 2 =$ probability that a 30-year-old selected at age 28 dies before age 31

$q_{[30]} \leq q_{[29]+1}$ because more recent info is more valuable

$q_{[27]+3} = q_{30}$ for a 3-year select period

${}_{2|}q_{[31]+1} = p_{[31]+1} \cdot p_{[31]+2} \cdot q_{34}$ (3-year select period)

Read across, then down

Linear Interpolation

$$f(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

Geomteric Series

$$a + ar + ar^2 + \ldots + ar^n = a\left(\frac{1 - r^{n+1}}{1 - r}\right)$$

Interest Theory Review

Interest

$$i = \frac{d}{1-d}$$

$$d = \frac{i}{1+i} = 1 - v$$

$$\delta = \ln(1+i)$$

$$(1+i)^n = e^{n\delta}$$

$$v = e^{-\delta}$$

$$v^n = \frac{1}{(1+i)^n} = e^{-n\delta}$$

$$1+i = \left(1 + \frac{i^{(m)}}{m}\right)^m$$

$$1-d = \left(1 - \frac{d^{(m)}}{m}\right)^m$$

Double the force of interest

$$\delta \to 2\delta$$

$$1+i \to (1+i)^2$$

$$v \to v^2$$

$$i \to 2i + i^2$$

$$d \to 2d - d^2$$

$$\frac{i}{\delta} \to \frac{2i+i^2}{2\delta}$$

Annuities

$$a_{\overline{n}|} = \frac{1-v^n}{i}$$

$$\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d}$$

$$\bar{a}_{\overline{n}|} = \frac{1-v^n}{\delta}$$

$$s_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$$

$$\ddot{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{d}$$

$$\bar{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{\delta} = \frac{e^{n\delta} - 1}{\delta}$$

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

$$(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$$

$$(I\bar{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{\delta}$$

$$(\bar{I}\bar{a})_{\overline{n}|} = \frac{\bar{a}_{\overline{n}|} - nv^n}{\delta}$$

$$(Da)_{\overline{n}|} = \frac{n - a_{\overline{n}|}}{i}$$

$$(\bar{D}\bar{a})_{\overline{n}|} = \frac{n - \bar{a}_{\overline{n}|}}{\delta}$$

Integrals

$$\int_0^\infty v^t\,dt = \bar{a}_{\overline{\infty}|} = \frac{1}{\delta}$$

$$\int_0^\infty t v^t\,dt = (\bar{I}\bar{a})_{\overline{\infty}|} = \frac{1}{\delta^2}$$

$$\int_0^n t v^t\,dt = (\bar{I}\bar{a})_{\overline{n}|}$$

$$\int_0^n (n-t) v^t\,dt = (\bar{D}\bar{a})_{\overline{n}|}$$

Normal Approximation

$$Y = X_1 + X_2 + X_3 + \ldots + X_n \quad X_i \text{ i.i.d. with } (\mu_x, \sigma_x^2)$$

$$Y \sim N(n\mu_x, n\sigma_x^2)$$

$$\Pr(Y \le y) = 0.95$$

$$\Pr\left(\frac{Y-\mu_y}{\sigma_y} \le \frac{y-\mu_y}{\sigma_y}\right) = 0.95$$

$$\Phi\left(\frac{y-\mu_y}{\sigma_y}\right) = 0.95$$

$$\frac{y-\mu_y}{\sigma_y} = \Phi^{-1}(0.95)$$

$$y = \mu_y + \sigma_y \cdot 1.645$$

Bernoulli Shortcut

$$Y = \begin{cases} a & p(a) = p \\ b & p(b) = 1 - p = q \end{cases}$$

$$Y = (a-b)X + b$$

$$Var(Y) = (a-b)^2 \cdot p \cdot q$$

Law of Total Probability

$$P(A) = P(A/B) \cdot P(B) + P(A/B') \cdot P(B')$$

Insurance

Whole Life Insurance: $E\left(Z^n\right)=\int_0^\infty b_t^n \cdot v_t^n \cdot {}_t p_x \cdot \mu_{x+t} dt$

The variance on a benefit of 1000 is 1000^2 times the variance on a benefit of 1.

The variance on a benefit of 1 on n insureds is only n times the variance of a benefit of 1 on one insured.

Variance and Covariance

$$Var(aX)=a^2 Var(X)$$
$$Var(aX+b)=a^2 Var(X)$$

If $E(A)=E(B)+E(C)$

Then $Var(A)=Var(B)+Var(C)+2Cov(B,C)$

$$=Var(B)+Var(C)+2\left[E(BC)-E(B)\cdot E(C)\right]$$

$$Cov(A,B)=\underbrace{E(AB)}_{0\text{ if Mut. Exclusive}}-E(A)\cdot E(B)$$

$$Cov(X,X)=Var(X)$$
$$Cov(X,Y+Z)=Cov(X,Y)+Cov(X,Z)$$

Variance of Endowment Insurance

$Z_1 \rightarrow$ term insurance

$Z_2 \rightarrow$ pure endowment

$Z_3 \rightarrow$ endowment insurance

$$\mathrm{Var}[Z_3]=\mathrm{Var}[Z_1]+\mathrm{Var}[Z_2]-2E[Z_1]E[Z_2]$$

$$ {}_5^2E_{45}={}_5E_{45}\cdot v^5$$

Increasing Insurance

$$\int_0^\infty t^n e^{-\delta t} dt=\frac{n!}{\delta^{n+1}}$$

Mixtures

You can weight expected values and 2nd moments, but NOT variance and NOT premium

Insurance Payable at Moment of Death Part 1

Name	Benefit	E(Z)	$E(Z^2)$
Whole Life Insurance	1	$\int_0^\infty v^t\, {}_tp_x \mu(x+t)dt$	$\int_0^\infty v^{2t}\, {}_tp_x \mu(x+t)dt$
N-Year Term	1	$\int_0^n v^t\, {}_tp_x \mu(x+t)dt$	$\int_0^n v^{2t}\, {}_tp_x \mu(x+t)dt$
N-Year Deferred Whole Life	1	$\int_n^\infty v^t\, {}_tp_x \mu(x+t)dt$	$\int_n^\infty v^{2t}\, {}_tp_x \mu(x+t)dt$
N-year Pure Endowment (not insurance)	1	$\int_0^n v^t\, {}_tp_x \cdot \mu(x+t)dt + v^n\, {}_np_x$	$\int_0^n v^{2t}\, {}_tp_x \cdot \mu(x+t)dt + v^{2n}\, {}_np_x$
N-year Endowment Insurance	1		
Whole Life Increasing Continuously	t	$\int_0^\infty tv^t\, {}_tp_x \cdot \mu(x+t)dt$	$\int_0^\infty t^2v^{2t}\, {}_tp_x \cdot \mu(x+t)dt \neq {}^2(\overline{I}\overline{A})_x$
Whole Life Increasing Annually	$\lfloor T+1 \rfloor$	$\int_0^\infty \lfloor t+1 \rfloor v^t\, {}_tp_x \cdot \mu(x+t)dt$	
N-Year Term Increasing Continuously	t		
Whole Life Decreasing Continuously	n - t		

Insurance Payable at Moment of Death Part 2

Name	Z (PV benefit)	E(Z)	E(Z) alternative	$E(Z^2)$	Var(Z)	If i=0
Whole Life Insurance	$Z = v^T$	$\bar{A}_x$	$\bar{A}^{1}_{x:\overline{n}\rvert} + {}_{n\rvert}\bar{A}_x$	${}^2\bar{A}_x$ ${}^2\bar{A}^{1}_{x:\overline{n}\rvert} + {}^2_{n\rvert}\bar{A}_x$	${}^2\bar{A}_x - (\bar{A}_x)^2$	1
N-Year Term	$Z = \begin{cases} v^T & 0 \le T \le n \\ 0 & T > n \end{cases}$	$\bar{A}^{1}_{x:\overline{n}\rvert}$	$\bar{A}_{x:\overline{n}\rvert} - {}_nE_x$	${}^2\bar{A}^{1}_{x:\overline{n}\rvert}$	${}^2\bar{A}^{1}_{x:\overline{n}\rvert} - (\bar{A}^{1}_{x:\overline{n}\rvert})^2$	${}_nq_x$
N-Year Deferred Whole Life	$Z = \begin{cases} 0 & 0 \le T \le n \\ v^T & T > n \end{cases}$	${}_{n\rvert}\bar{A}_x$	${}_nE_x \cdot \bar{A}_{x+n}$	${}^2_{n\rvert}\bar{A}_x$	${}^2_{n\rvert}\bar{A}_x - ({}_{n\rvert}\bar{A}_x)^2$	${}_np_x$
N-year Pure Endowment (not insurance)	$Z = \begin{cases} 0 & 0 \le T \le n \\ v^n & T > n \end{cases}$	$\bar{A}_{x:\overline{n}\rvert}^{\;\;1}$	${}_nE_x$ $v^n \cdot {}_np_x$	${}^2\bar{A}_{x:\overline{n}\rvert}^{\;\;1}$ ${}^2_nE_x$ $v^{2n} \cdot {}_np_x$	${}^2_nE_x - ({}_nE_x)^2$ $v^{2n}\ {}_np_x \cdot {}_nq_x$	${}_np_x$
N-year Endowment Insurance	$Z = \begin{cases} v^T & 0 \le T \le n \\ v^n & T > n \end{cases}$	$\bar{A}_{x:\overline{n}\rvert}$	$\bar{A}^{1}_{x:\overline{n}\rvert} + {}_nE_x$	${}^2\bar{A}_{x:\overline{n}\rvert}$ ${}^2\bar{A}^{1}_{x:\overline{n}\rvert} + {}^2_nE_x$	${}^2\bar{A}_{x:\overline{n}\rvert} - (\bar{A}_{x:\overline{n}\rvert})^2$	1
Deferred Term Insurance		${}_{m\rvert n}\bar{A}_x$	$\bar{A}_x({}_mE_x - {}_{m+n}E_x)$			
Whole Life Increasing Continuously	$Z = Tv^T$	$(\bar{I}\bar{A})_x$				$\mathring{e}_x$
N-Year Term Increasing Continuously	$Z = \begin{cases} Tv^T & 0 \le T \le n \\ 0 & T > n \end{cases}$	$(\bar{I}\bar{A})^{1}_{x:\overline{n}\rvert}$				
Whole Life Decreasing Continuously	$Z = \begin{cases} (n-T)v^T & 0 \le T \le n \\ 0 & T > n \end{cases}$	$(\bar{D}\bar{A})^{1}_{x:\overline{n}\rvert}$				

Insurance Payable at End of Year of Death

Name	Z (PV benefit)	E(Z)	E(Z) alternative	$E(Z^2)$	Var(Z)
Discrete Whole Life	$Z = v^{k+1} \quad k = 0,1,2,\ldots$	A_x	$A^{1}_{x:\overline{n}\rvert} + {}_{n\rvert}A_x$	${}^2A_x$	${}^2A_x - (A_x)^2$
N-Year Term	$Z = \begin{cases} v^{k+1} & k = 0,1,2,\ldots \\ 0 & k = n, n+1,\ldots \end{cases}$	$A^{1}_{x:\overline{n}\rvert}$		${}^2A^{1}_{x:\overline{n}\rvert}$	${}^2A^{1}_{x:\overline{n}\rvert} - \left(A^{1}_{x:\overline{n}\rvert}\right)^2$
N-Year Deferred Whole Life	$Z = \begin{cases} 0 & k = 0,1,\ldots,n-1 \\ v^{k+1} & k = n, n+1,\ldots \end{cases}$	${}_{n\rvert}A_x$	${}_nE_x A_{x+n}$	${}^2_{n\rvert}A_x$	${}^2_{n\rvert}A_x - \left({}_{n\rvert}A_x\right)^2$
N-Year Deferred M-Year Term	$Z = \begin{cases} 0 & k = 0,1,\ldots,n-1 \\ v^{k+1} & k = n, n+1,\ldots,n+m-1 \\ 0 & k = n+m, n+m+1,\ldots \end{cases}$	${}_{n\rvert m}A_x$		${}^2_{n\rvert m}A_x$	${}^2_{n\rvert m}A_x - \left({}_{n\rvert m}A_x\right)^2$
N-year Endowment Insurance	$Z = \begin{cases} v^{k+1} & k = 0,1,\ldots,n-1 \\ v^{n} & k = n, n+1,\ldots, \end{cases}$	$A_{x:\overline{n}\rvert}$	$A^{1}_{x:\overline{n}\rvert} + {}_nE_x$	${}^2A_{x:\overline{n}\rvert}$	${}^2A_{x:\overline{n}\rvert} - \left(A_{x:\overline{n}\rvert}\right)^2$
Whole Life Increasing	$Z = (k+1)v^{k+1}$	$(IA)_x$		$\neq {}^2(IA)_x$	
Increasing Term Insurance		$(IA)^{1}_{x:\overline{n}\rvert}$			
Decreasing Term Insurance		$(DA)^{1}_{x:\overline{n}\rvert}$			

Insurance Under Special Laws

Insurance	Constant Force	DeMoivre's Law
${}_nE_x$	$e^{-n(\mu+\delta)}$	$e^{-n\delta}\left(\dfrac{\omega-x-n}{\omega-x}\right)$
$\bar{A}_x$	$\dfrac{\mu}{\mu+\delta}$	$\dfrac{\bar{a}_{\overline{\omega-x}\rceil}}{\omega-x}$
A_x	$\dfrac{q}{q+i}$	$\dfrac{a_{\overline{\omega-x}\rceil}}{\omega-x}$
${}^2\bar{A}_x$	$\dfrac{\mu}{\mu+2\delta}$	$\dfrac{\bar{a}_{\overline{\omega-x}\rceil}}{\omega-x} @ i^*=2i+i^2$
${}^2A_x$	$\dfrac{q}{q+2i+i^2}$	$\dfrac{a_{\overline{\omega-x}\rceil}}{\omega-x} @ i^*=2i+i^2$
$\bar{A}^1_{x:\overline{n}\rceil}$	$\bar{A}_x\left(1-{}_nE_x\right)$	$\dfrac{\bar{a}_{\overline{n}\rceil}}{\omega-x}$
$A^1_{x:\overline{n}\rceil}$	$A_x\left(1-{}_nE_x\right)$	$\dfrac{a_{\overline{n}\rceil}}{\omega-x}$
${}_{n\|}\bar{A}_x$	$\bar{A}_x \cdot {}_nE_x$	$e^{-n\delta}\left(\dfrac{\bar{a}_{\overline{\omega-x-n}\rceil}}{\omega-x}\right)$
${}_{m\|n}\bar{A}_x$	$\bar{A}_x\left({}_mE_x-{}_{m+n}E_x\right)$	
$(\bar{I}\bar{A})_x$	$\dfrac{\mu}{(\mu+\delta)^2}$	$\dfrac{(\bar{I}\bar{a})_{\overline{\omega-x}\rceil}}{\omega-x}$
$(IA)_x$	$A_x\left(\dfrac{1+i}{q+i}\right)$	$\dfrac{(Ia)_{\overline{\omega-x}\rceil}}{\omega-x}$
$(\bar{I}\bar{A})^1_{x:\overline{n}\rceil}$		$\dfrac{(\bar{I}\bar{a})_{\overline{n}\rceil}}{\omega-x}$
$(IA)^1_{x:\overline{n}\rceil}$		$\dfrac{(Ia)_{\overline{n}\rceil}}{\omega-x}$

Relationship between Discrete and Continuous

Assume UDD

$$\bar{a}_{\overline{n}|} = \frac{i}{\delta} a_{\overline{n}|}$$

$$\bar{A}_x \geq A_x$$

$$\bar{A}_x = \frac{i}{\delta} A_x$$

$$^2\bar{A}_x = \frac{2i + i^2}{2\delta} \, {}^2A_x$$

$$\bar{A}^{1}_{x:\overline{n}|} = \frac{i}{\delta} A^{1}_{x:\overline{n}|}$$

$$\bar{A}_{x:\overline{n}|} = \frac{i}{\delta} A^{1}_{x:\overline{n}|} + {}_nE_x$$

$$\left(I\bar{A}\right)_x = \frac{i}{\delta}(IA)_x$$

$$\left(I\bar{A}\right)^{1}_{x:\overline{n}|} = \frac{i}{\delta}(IA)^{1}_{x:\overline{n}|}$$

$$\left(D\bar{A}\right)^{1}_{x:\overline{n}|} = \frac{i}{\delta}(DA)^{1}_{x:\overline{n}|}$$

$$\left(\bar{I}\bar{A}\right)_x = \frac{i}{\delta}(IA)_x - \frac{i}{\delta}\left(\frac{1}{d} - \frac{1}{\delta}\right)A_x$$

$\left(\bar{\mathrm{I}}\bar{\mathrm{A}}\right)^{1}_{x:\overline{n}|}$

A → life insurance

$\bar{\mathrm{A}}$ → paid at instant of death

I → increasing

$\bar{\mathrm{I}}$ → increasing continuously

1 → term insurance

Formulas to Know

$$A_x = vq_x + vp_x A_{x+1}$$

$$A_x = vq_x + v^2 {}_{1|}q_x + v^2 {}_2p_x A_{x+2}$$

$$A^{1}_{x:\overline{n}|} = vq_x + vp_x A^{1}_{x+1:\overline{n-1}|}$$

$$^2\bar{A}_x = {}^2\bar{A}^{1}_{x:\overline{n}|} + {}^2_{n|}\bar{A}_x$$

$$^2\bar{A}_{x:\overline{n}|} = {}^2\bar{A}^{1}_{x:\overline{n}|} + {}^2_nE_x$$

$$(IA)_x = A_x + vp_x (IA)_{x+1}$$

$$(IA)_x = vq_x + vp_x \left(A_{x+1} + (IA)_{x+1}\right)$$

$$(IA)^{1}_{x:\overline{n}|} = A^{1}_{x:\overline{n}|} + vp_x (IA)^{1}_{x+1:\overline{n-1}|}$$

$$(IA)^{1}_{x:\overline{n}|} = vq_x + vp_x \left(A^{1}_{x+1:\overline{n-1}|} + (IA)^{1}_{x+1:\overline{n-1}|}\right)$$

$$(DA)^{1}_{x:\overline{n}|} = nA^{1}_{x:\overline{1}|} + vp_x (DA)^{1}_{x+1:\overline{n-1}|}$$

$$(DA)^{1}_{x:\overline{n}|} = nvq_x + vp_x (DA)^{1}_{x+1:\overline{n-1}|}$$

$$(\bar{I}\bar{A})^{1}_{x:\overline{n}|} + (\bar{D}\bar{A})^{1}_{x:\overline{n}|} = n\bar{A}^{1}_{x:\overline{n}|}$$

$$(I\bar{A})^{1}_{x:\overline{n}|} + (D\bar{A})^{1}_{x:\overline{n}|} = (n+1)\bar{A}^{1}_{x:\overline{n}|} \text{ (level term insurance of 'n')}$$

$$(IA)^{1}_{x:\overline{n}|} + (DA)^{1}_{x:\overline{n}|} = (n+1)\, A^{1}_{x:\overline{n}|}$$

Note: Recursive formulas also work for second moments