Practice Exam M

by

G.V. Ramanathan

Version 2.0: Spring 2005

A note to the student

I have updated the practice exam to reflect the change in syllabus from Course 3 to Exam M.

The purpose of this practice exam is to help you review some of the important concepts and techniques needed for the SOA Exam M. It is not intended as a drill exam. Please do not race through it. The focus should be on the process of setting up and solving each problem rather than on speed. Some of the problems may perceived as hard. But each is designed to reinforce a concept that is essential for the SOA Exam M. I believe that thinking through such problems can help one diagnose the areas that need work and help one prepare better for the SOA exam.

Even though I have gone through the manuscript several times, it is possible that some errors might have escaped my notice. If you happen to find any, I should appreciate it if you would let me know by sending a note to gvr@actuarialexamprep.us.

For updates and corrections please visit www.actuarialexamprep.us or www.actuary.ca.

I wish you all the best.

G.V. Ramanathan

Other study material:

- G.V. Ramanathan is the author of a Study Guide for Exam M that is self-contained and streamlined and can be used as a textbook. It is available through your actuarial bookstore. You can find a sample chapter on the web site www.actuarialexamprep.us
- Each exam term G.V. Ramanathan also offers online courses for Exam M. Details are available at www.actuarialexamprep.us.

About the author: G.V. Ramanathan, Ph.D., A.S.A. is Professor Emeritus in the Department of Mathematics, Statistics and Computer Science at the University of Illinois at Chicago. He has taught actuarial review courses for 15 years. He has been teaching online courses for actuarial exams since 1999 and has published study guides for Exams P and M and for the old Courses 1, 3 and 151.

Practice Exam M

1. A company has started an internship program by hiring 120 interns on January 1, 2005. During each year several of the interns will quit. At the beginning of every year the company will hire new interns to replace those who have left and no more. The following table is representative of the number of interns, l_x, that will continue to work for the company at time x years, starting from a closed population of 1000 new hires.

l_0	1,000
l_1	700
l_2	400
l_3	200
l_4	0

Calculate the expected number of interns that the company will hire on January 1, 2008.

(A) 41 **(B)** 43 **(C)** 45 **(D)** 47 **(E)** 49

2. Let ${}_t\overline{V}_x$ stand for the benefit reserve at time $t > 0$ for a fully continuous whole life insurance of 1 on (x). Which of the following statements are true?

I. If mortality for (x) follows De Moivre's law then ${}_t\overline{V}_x > 0$

II. If the future lifetime of (x) has a Pareto distribution, then ${}_t\overline{V}_x > 0$

III. If the hazard rate is proportional to the age, then ${}_t\overline{V}_x > 0$.

(A) I and II only **(B)** I and III only **(C)** II and III only **(D)** I, II and III **(E)** The correct answer is not given by A, B, C or D.

3. A machine was purchased on January 1, 2000. It will be replaced by a new machine the moment it fails or on January 1, 2012, whichever is earlier. The hazard rate for the machine is 0.1 per year. The machine is functioning on January 1, 2002. Calculate the expected time (in years from January 1, 2002) when the machine will be replaced.

   (A) 6.3 **(B)** 7.4 **(C)** 8.5 **(D)** 9.6 **(E)** 10.0

4. Two computers are on the same network. Their future lifetimes are dependent only through an exponential common shock variable with intensity $\lambda = 0.025$. The marginal distribution of the future lifetime of Computer I is exponential with mean 10. The marginal distribution of the future lifetime of Computer II is exponential with mean 8. Calculate the probability that the future lifetime of Computer I will be at least as long as that of Computer II.

   (A) 3/8 **(B)** 1/2 **(C)** 5/8 **(D)** 3/4 **(E)** 7/8

5. Jack's professor has offered him a nonrenewable research fellowship for one year. If funds are cut during the year at time t, the professor will terminate the fellowship at t. The probability that by time t the funds will be cut is $0.1t,\ 0 < t < 1$. If the funds are not cut the professor will terminate the fellowship at the end of one year.

   It is also in Jack's nature that, independently of what he undertakes, the chances that he will quit during the year is 20%. If he quits, the time of quitting will be uniformly distributed over the year. There are no other decrements.

   Calculate the probability that Jack's fellowship will come to an end because the professor has terminated it.

   (A) 0.1 **(B)** 0.2 **(C)** 0.3 **(D)** 0.8 **(E)** 0.9

Use this information for Questions 6 and 7:

Melissa buys a desktop computer and a laptop computer at the same time. At the moment when both have broken down or at the end of 4 years, whichever comes first, Melissa will buy a fancy computer system for 3000 dollars. For that purpose Melissa will save money at a continuous constant annual rate of P. You are given:

- The future lifetimes of the desktop and laptop are independent.
- The future lifetime of the desktop is uniformly distributed over $(0, 3)$ and the future lifetime of the laptop is uniformly distributed over $(0, 5)$.
- $\delta = 0.04$.

6. Calculate P such that the actuarial present value of the amount that Melissa has saved will equal the actuarial present value of the 3000 dollars she will pay for the system.

   (A) 1000 **(B)** 1100 **(C)** 1200 **(D)** 1300 **(E)** 1400

7. Calculate the minimum P such that the probability that the accumulated value of the amount that Melissa has saved will cover the cost (3000) of the new system with a probability of at least 75%.

   (A) 740 **(B)** 850 **(C)** 1100 **(D)** 1370 **(E)** 1490

8. A two-year dental insurance policy will reimburse 100% of the cost of dental work during each year. The reimbursement will be made at the end of the year. The dental costs in the first year as well as in the second year are uniformly distributed over $(0, 1200)$ with a covariance of 100,000. $i = 0.06$. Assuming that the policy will be in force for two years, calculate the standard deviation of the present value of the payments.

   (A) 600 **(B)** 608 **(C)** 612 **(D)** 616 **(E)** 620

9. The annual benefit premium rate for a fully continuous whole life insurance of 1000 on (x) is 10. At the time of issue the policy is modified to pay a benefit of 10,000 if the death is due to accident and 1,000 if death is due to non-accidental causes. The force of mortality for accidental death is $\mu^{(a)}(x)$ and the force of mortality for non-accidental death is $\mu^{(n)}(x)$. You are given that $\mu^{(a)}(x) = (1/9)\mu^{(n)}(x)$. Calculate the increase in the annual benefit premium rate.

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

10. Peter aged 30 and Janice aged 40 are independent lives. A deferred continuous annuity will pay at an annual rate of 1 as long as Peter or Janice is living after the age of 65. The force of mortality for Peter is 0.02, the force of mortality for Janice is 0.04 and the force of interest is 0.06. Calculate the actuarial present value of this annuity.

    (A) 1.5 **(B)** 1.6 **(C)** 1.7 **(D)** 1.8 **(E)** 1.9

11. Mr. Star wants to purchase a whole life insurance but he refuses to give his date of birth. FBN insurance company offers a policy that will, in return for a single benefit premium of $625,000$, pay 1,000,000 at the moment of Mr. Star's death. FBN assumes for its calculation that the future lifetime of Mr. Star is exponentially distributed with mean $1/M$ and M is a random variable that is uniformly distributed over $(\delta, 2\delta)$, where δ is the force of interest.

    Calculate the relative security loading that FBN charges over the benefit premium.

    (A) 0.05 **(B)** 0.1 **(C)** 0.15 **(D)** 0.20 **(E)** 0.25

12. There are n identical, independent, fully continuous whole life policies, each of benefit 1 in a portfolio. The force of mortality is 0.01 for each life. The force of interest is δ. $P(n)$ is the smallest level premium per policy such that in the normal approximation, the probability of a positive aggregate loss is at most 0.05. Calculate $\lim_{n\to\infty} P(n)$.

   (A) 0 **(B)** 0.01 **(C)** 0.015 **(D)** 0.02 **(E)** 0.25

13. The future lifetimes of (x) and (y) are independent. The force of mortality is constant for both lives. $\mu_x = 0.02$ and $\mu_y = 0.03$.

   Calculate the expected duration between the first death and the second death.

   (A) 41.7 **(B)** 42.5 **(C)** 43.3 **(D)** 44.1 **(E)** 44.9.

14. An insurance policy will cover theft of a car for two years.

   - If the car is stolen in the first year, the policy will pay $20,000.
   - If the car is stolen during the second year the policy will pay $12,000.
   - If the car is not stolen by the end of the second year the insurer will return 10% of the total of the premiums paid without interest. (This payment will be made at the end of the second year.)
   - Benefits will be paid at the end of the year.
   - Level premiums are paid at the beginning of each year.
   - Premium is determined by the equivalence principle.
   - $i = 0.06$.
   - The probability that the car will be stolen in each year is 0.1. (i.e., $q_x = 0.1$ and $q_{x+1} = 0.1$.)

   Calculate the benefit reserve at the end of the first year.

   (A) -255 **(B)** -250 **(C)** -245 **(D)** 250 **(E)** 255

15. A math professor has received a grant to work on a project that involves proving two theorems. The professor will prove one of the theorems before he starts work on the other. You are given

- The amount of time it takes to prove one theorem is independent of the time for the other.
- The amount of time to prove each theorem is exponentially distributed with mean 1 year.
- The professor will receive funding at a continuous annual rate of 242,000 dollars per year as long as it takes him to prove the two theorems.
- The force of interest is 0.1.

Calculate the actuarial present value of the amount of the grant.

(A) 234,000 **(B)** 298,000 **(C)** 356,000 **(D)** 420,000 **(E)** 484,000

16. If we assume uniform distribution of deaths over each year of age, which of the following statements are true in general?

I. $\overset{\circ}{e}_x = p_x \left(\overset{\circ}{e}_{x+1} + 1/2 \right)$.

II. $\overline{A}_{xy} = \frac{i}{\delta} A_{xy}$

III. $\overline{A}_{x:\overline{n|}} = \frac{i}{\delta} A_{x:\overline{n|}}$.

(A) I only **(B)** II only **(C)** III only **(D)** None **(E)** The correct answer is not given by A, B, C or D.

17. Aggregate claims has a compound Poisson distribution with expected number of claims 1 and individual claim amounts 1 and 2. With a deductible of 1 on the aggregate claims, the net stop-loss premium is 0.867879. Calculate the reduction in the net stop-loss premium if the deductible is raised to 1.5.

(A) 0.21 **(B)** 0.22 **(C)** 0.23 **(D)** 0.24 **(E)** 0.25

18. For a fully discrete whole life insurance of 10 on (x) you are given:

- ${}_{k|}q_x = 0.2, \; k = 0, 1, 2, 3, 4.$
- $v = 0.8.$
- The benefit reserve at the end of 4 years is 5.672.

Calculate $Var({}_3L|K(x) \geq 3)$.

(A) 1.5 **(B)** 2.0 **(C)** 2.5 **(D)** 3.0 **(E)** 3.5

19. Which of the following statements are true?

I. For a Pareto distribution with finite mean, the median is larger than the mean.

II. If a loss has a Pareto distribution with finite mean, for a fixed positive ordinary deductible, the loss elimination ratio increases with α.

III. If the future lifetime random variable has a Pareto distribution, the hazard rate decreases with age.

(A) I and II only **(B)** II and III only **(C)** I and III only **(D)** I, II and III **(E)** The correct answer is not given by A, B, C or D.

.

20. A loss, X has the following distribution:

x	$Pr(X = x)$
0	0.60
100	0.15
300	0.10
400	0.10
600	0.03
800	0.02

An ordinary deductible is imposed. The expected payment per loss is 74. Calculate the loss elimination ratio.

(A) 0.08 **(B)** 0.18 **(C)** 0.28 **(D)** 0.38 **(E)** 0.48

21. This year losses have an exponential distribution with mean 100. Because of inflation the losses next year will be 10% higher. Next year a deductible of 10 will be imposed. Calculate the probability that the cost per payment next year will be less than 80.

(A) 0.48 **(B)** 0.50 **(C)** 0.52 **(D)** 0.54 **(E)** 0.56

22. A random loss has a Pareto distribution with $\alpha = 3$ and θ a random variable that is uniformly distributed over the interval $(100, 200)$. Calculate the variance of the loss.

(A) Between 4,000 and 8,000 **(B)** Between 8,000 and 12,000 **(C)** Between 12,000 and 16,000 **(D)** Between 16,000 and 20,000 **(E)** Over 20,000

23. The random variable N has a $(a, b, 0)$ distribution. That means

$$Pr(N = n) = \left(a + \frac{b}{n}\right) Pr(N = n - 1),\ n = 1, 2, \cdots.$$

You are given that

- $Pr(N = 2|N > 0) = 1/4$
- $Pr(N = 3|N > 0) = 1/6$
- $Pr(N = 4|N > 0) = 5/48.$

Calculate $Pr(N > 1)$.

(A) 0.50 **(B)** 0.55 **(C)** 0.60 **(D)** 0.65 **(E)** 0.70

24. A professor is collecting questions for Exam-M by looking through a large pile papers. On each sheet of paper there is exactly one question. On the average, ten percent of the questions are related to Exam-M. Of the rest, which are unrelated to Exam-M, one-third of the questions are on topology. The professor picks one sheet at a time at random. Calculate the probability that the professor will get at least one topology question before he gets the third question for Exam-M.

(A) 0.90 **(B)** 0.92 **(C)** 0.94 **(D)** 0.96 **(E)** 0.98

25. The number of accidents over a period has a Poisson distribution with mean 2. For each accident there can be 0, 1, 2 or 3 claims with equal probability. The number of claims for each accident and the number of accidents are all independent. Calculate the probability that there will be at least two claims in a given period.

(A) 0.34 **(B)** 0.42 **(C)** 0.50 **(D)** 0.58 **(E)** 0.66

26. The number of claims is 0, 1 or 2 with equal probability. The individual claim amount is uniformly distributed over the interval $(100, 500)$. The number of claims and the claim amounts are all independent. If the aggregate claims is less than 300 then ten percent of the difference will be paid as a dividend. Calculate the expected dividend.

(A) 11.5 **(B)** 11.6 **(C)** 11.7 **(D)** 11.8 **(E)** 11.9

27. The expense-loaded level annual premium for a fully discrete whole life insurance of 1,000 on (40) is calculated using the equivalence principle and the ILT with $i = 0.06$. You are given:

- Selling commission is 30% of the expense-loaded premium in the first year
- Renewal commissions are 5% of the expense-loaded premium for the second to the tenth year
- The expenses are 10 in the first year and 4 thereafter
- Taxes are 2% of the expense-loaded premium each year
- Expenses, commission and taxes are paid at the beginning of each year

Calculate the expense-loaded annual premium.

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

28. Jim is now 35. When he attains the age of 65 he will start receiving an annuity at the rate of 10,000 per year paid continuously as long as he is alive. If Jim dies between 5 years and 30 years from now a death benefit of 50,000 will be paid at the moment of his death. There will be no payment if death occurs within 5 years. You are given:

- The force of interest is 0.05.
- The future lifetime of Jim has an exponential distribution with mean 20.

Calculate the 90th percentile of the present value of payment.

(A) 36,000 **(B)** 37,000 **(C)** 38,000 **(D)** 39,000 **(E)** 40,000

29. The distribution of aggregate claims is compound Poisson with $\lambda = 4$. The individual claim amount is uniformly distributed over $(0, 1000)$. Let M be the number of claims that exceed the amount of 100 and N be the number that do not exceed 100. Calculate $E\left\{\frac{M}{N+1}\right\}$.

(A) 2.0 **(B)** 2.5 **(C)** 3.0 **(D)** 3.5 **(D)** 4.0

30. Mortality for a newborn follows De Moivre's law with $\omega = 100$. Let $Z(x)$ be the amount of time lived by the new born in the first x years and $W(x)$ be the amount of time lived by the newborn after x years. Calculate the maximum value of $Cov(Z(x), W(x))$. (Units of time are years.)

(A) 156 **(B)** 157 **(C)** 158 **(D)** 159 **(E)** 160

31. There are 100 independent losses, identically distributed with a continuous density. For each individual loss an insurance will pay the excess of the loss over d if the loss exceeds d and 25% of the excess of d over the loss if d exceeds the loss.

Let d_m be the value of d that minimizes the expected payment per loss. Calculate the expected number of losses that exceed d_m.

(A) 15 **(B)** 20 **(C)** 25 **(C)** 30 **(D)** 35 **(E)** 40

32. Which of the following statements are true?

I. The inverse exponential distribution with $\theta = 1$ has a heavier tail than the Pareto distribution with $\alpha = 1$ and $\theta = 1$.

II. If the future lifetime of (x) is uniformly distributed over $(0, A)$ then $e_x = 0.5(A - 1)$.

III. If the future lifetime at birth has a Pareto distribution with finite mean, then the complete-expectation-of-life increases with age.

(A) None **(B)** I only **(C)** II only **(D)** III only **(E)** The correct answer is not given by A, B, C or D.

.

33. The future lifetimes of (x), (y) and (z) are independent and each is exponentially distributed with mean 10. Calculate the probability that the last death will occur before the 90-th percentile of the first death.

(A) 0.05 **(B)** 0.10 **(C)** 0.15 **(D)** 0.20 **(E)** 0.25

34. Mortality for independent lives (50) and (60) follows De Moivre's law with $\omega = 100$. Calculate the probability that they die within 10 years of each other and the average of their ages at death is between 75 and 80.

(A) 0.02 **(B)** 0.05 **(C)** 0.08 **(D)** 0.09**(E)** 0.10

35. Jean, who is preparing for Exam 3, has left it to higher powers to dictate her study habits. Every day, she will toss a fair coin. If the result is heads she will study that day. If the result is tails she will not study that day. If the result is heads, in order to determine the number of hours she will study, she will roll a fair die. The number of hours she will study will be the number on the face that turns up on the throw of the die.

Jean will do this five days a week for 16 weeks. Assuming that the tosses and the throws of the die are all independent and using the normal approximation without the continuity correction, the probability that Jean will have studied at least 120 hours over the 16 weeks is $\Phi(c)$. Calculate c.

(A) 1.00 **(B)** 1.05 **(C)** 1.10 **(D)** 1.15 **(D)** 1.20 **(E)** 1.25

36. In a compound process the claim number process is Poisson with rate 1 per unit time. The individual claim amount is exponentially distributed with mean 3. The claim amounts and the number of claims are all independent. Let $S(t)$ be the aggregate claims up to time t and $A = (1/3)\{S(1) + S(2) + S(3)\}$. Calculate $Var(A)$.

(A) 24 **(B)** 26 **(C)** 28 **(D)** 30 **(E)** 32

37. The number of claims has a geometric distribution with mean 2. The individual claim amount distribution is Pareto with $\theta = 1000$ and $\alpha = 2$. The number of claims and the claim amounts are all independent. Calculate the probability that the largest claim is less than 1000.

(A) 1/4 **(B)** 1/2 **(C)** 2/3 **(D)** 3/4 **(E)** 4/5

38. For independent lives (x) and (y), the force of mortality is 0.02 and 0.06 respectively. Calculate the expected duration between the their deaths.

(A) 39 **(B)** 40 **(C)** 41 **(D)** 42 **(E)** 43

39. An actuary had modeled fire damage to a garage with a Pareto distribution with $\alpha = 2$ and $\theta = 10,000$. The insurance company has been charging the owner of the garage a premium of 150% of the expected loss in return for reimbursement of the complete loss.

The insured complains that the loss can never exceed 30,000 and that he is paying for coverage of losses that he will never suffer.

The insurance company revises the model. The new distribution will still have the form of Pareto with $\alpha = 2$ and $\theta = 10,000$, that is, the PDF will be proportional to $(x + 10,000)^{-3}$, but only for losses up to 30,000. The probability of the loss being larger than 30,000 will be set to zero. The premium will still be 150% of the expected loss.

Calculate the decrease in the premium as a result of revision of the model.

(A) 400 **(B)** 1,000 **(C)** 2,000 **(D)** 4,000 **(E)** 6,000

40. There are three possible states for an insured: (1) well (2) disabled and (3) dead. For three years from the time of issue the transition matrices are given by

$$\mathbf{Q}_n = \begin{bmatrix} 0.8 - 0.1n & 0.1 + 0.05n & 0.1 + 0.05n \\ 0.6 - 0.1n & 0.3 - 0.1n & 0.1 + 0.2n \\ 0 & 0 & 1 \end{bmatrix}; n = 0, 1, 2.$$

You are given:

- At the beginning of each year that the insured is disabled, a benefit of 100 will be paid.
- A death benefit of 1,000 will be paid at the end of the year of death.
- At the beginning of each year, if and only if the insured is well, a premium of P will be collected.
- The contract will be terminated at the end of the third year.
- The premium is determined by the equivalence principle.
- At the time of issue the insured is well.
- $v = 0.8$.

Calculate P.

(A) 131 **(B)** 134 **(C)** 137 **(D)** 140 **(E)** 143

Answer Key

1.	E	21.	C
2.	B	22.	D
3.	A	23.	A
4.	C	24.	E
5.	D	25.	E
6.	B	26.	C
7.	E	27.	E
8.	B	28.	A
9.	E	29.	C
10.	A	30.	A
11.	A	31.	B
12.	B	32.	D
13.	C	33.	C
14.	A	34.	B
15.	D	35.	C
16.	D	36.	C
17.	B	37.	C
18.	D	38.	D
19.	B	39.	E
20.	D	40.	E

Solutions to Practice Exam

1. Denote by a_n the number of hires in year n. Then

$$\begin{aligned} a_0 &= 120 \\ a_1 &= 0.3\,a_0 = 36 \\ a_2 &= 0.3\,a_1 + 0.3\,a_0 = 46.8 \\ a_3 &= 0.3\,a_2 + 0.3\,a_1 + 0.2\,a_0 = (0.3)(46.8) + (0.3)(36) + 0.2(120) \\ &= 48.84. \end{aligned}$$

   Answer: E

2. If the force of mortality increases (decreases) with age, x, then

$$_tp_x = \exp\left\{-\int_0^t \mu(x+s)\,ds\right\}$$

   decreases (increases) with x,

$$\overline{a}_x = \int_0^\infty e^{-\delta t}\,{}_tp_x\,dt,$$

   is a decreasing (increasing) function of x and

$$_t\overline{V}_x = 1 - \frac{\overline{a}_{x+t}}{\overline{a}_x}$$

   is positive (negative). Thus if $\mu(x)$ is increasing then the benefit reserve at t is positive and if $\mu(x)$ is decreasing the benefit reserve is negative.

   If the survival function is Pareto, then the hazard rate is a decreasing function of age. In the other two cases it is an increasing function of age. Therefore for Pareto the benefit reserve is negative and in the other two cases it is positive. So I and III are true and II is false.

   Answer: B

3. If X is the future lifetime of the machine at age 2 and T the time of replacement measured from January 1, 2002, then

$$T = \begin{cases} X & \text{if } 0 < X \leq 10 \\ 10 & \text{if } X > 10. \end{cases} = E(X \wedge 10)$$

The pdf of X is $f(x) = 0.1e^{-0.1x}$ (the exponential distribution is memoryless). Therefore

$$\begin{aligned} E(T) &= E(X \wedge 10) = \int_0^{10} e^{-0.1x}\, dx \\ &= 10(1 - e^{-1}) \sim 6.32. \end{aligned}$$

Answer: A

4. Let $s_I(x)$ and $s_{II}(y)$ be the **marginal** survival functions. Then the joint survival function is

$$\begin{aligned} s(x,y) &= s_I(x)s_{II}(y)e^{\lambda x+\lambda y-\lambda \max(x,y)} \\ &= e^{-0.075x-0.1y-0.025\max(x,y)} \\ Pr(X > Y) &= \int_0^\infty \int_y^\infty \frac{\partial^2}{\partial x\, \partial y} s(x,y)dx\, dy \\ &= \int_0^\infty \int_y^\infty \frac{\partial^2}{\partial x\, \partial y} e^{-0.1x-0.1y}dx\, dy \ \ \text{(since in this region } x > y) \\ &= 1/2 \\ Pr(X = Y) &= \lambda \int_0^\infty s_I(t)s_{II}(t)e^{\lambda t}\, dt \\ &= 0.025 \int_0^\infty e^{-0.2t}\, dt = 1/8 \\ Pr(X \geq Y) &= 5/8. \end{aligned}$$

Answer: C

(Quite generally, if the **marginal** distributions are exponential with rates μ_I and μ_{II} then $Pr(X > Y) = (\mu_{II} - \lambda)(\mu_I + \mu_{II} - \lambda)^{-1}$ and $Pr(X \geq Y) = \mu_{II}(\mu_I + \mu_{II} - \lambda)^{-1}$.)

5. Let (1) be the cause that the professor terminates the fellowship and (2) the cause that Jack quits. Then we are given:

$$_tq_x'^{(1)} = 0.1t,\ 0 < t < 1,\ q_x'^{(1)} = 1;\ _tq_x'^{(2)} = 0.2t.$$

Therefore

$$q_x^{(2)} = 0.2\int_0^1 (1 - 0.1t)\,dt = 0.2(1 - 0.05) = 0.19.$$

Since $q_x^{(1)} + q_x^{(2)} = 1,\ q_x^{(1)} = 0.81$.

Answer: D

6. This is a four year endowment insurance on the last survivor. The APV is $3000\overline{A}_{\overline{xy}:\overline{4|}}$. The present value variable is $3000e^{-0.04T}$ if $T \leq 4$ and $3000\,e^{-0.16}$ if $T > 4$. The pdf of T is

$$f(t) = \begin{cases} \frac{2t}{15} & 0 < t \leq 3 \\ \\ \frac{1}{5} & 3 < t < 5 \end{cases}$$

and 0 elsewhere. Therefore

$$\overline{A}_{\overline{xy}:\overline{4|}} = (2/15)\int_0^3 e^{-0.04t}t\,dt + (1/5)\int_3^4 e^{-0.04t}\,dt + (1/5)e^{-0.16} = 0.89840.$$

(Integrate by parts to evaluate the above.)

The APV of saved amount is

$$P\,\overline{a}_{\overline{xy}:\overline{4|}} = P(1 - \overline{A}_{\overline{xy}:\overline{4|}})/0.04 = 2.54P.$$

This should equal 3000(0.8984). Hence $P = 1,061$.

Answer: B

7. Suppose T is the time of purchase of the new system. Then

$$L(T) = (3000 + 25P)e^{-0.04T} - 25P$$

is a decreasing function of T. If we set P such that $L(t) = 0$, then $L(T)$ will be negative if $T > t$. So we need to choose the largest t such that $Pr(T > t) \geq 0.75$. Hence t is the 25-th percentile of T.

From the last problem we have

$$F(t) = \begin{cases} \frac{t^2}{15} & 0 < t < 3 \\ \\ \frac{t}{5} & 3 < t < 5 \\ \\ 1 & t > 5 \end{cases}.$$

The 25-th percentile of T is $\sqrt{(15/4)}$. Therefore we set $L\{\sqrt{(15/4)}\} = 0$ or

$$P = 3000e^{-0.04\sqrt{(15/4)}} \frac{0.04}{1 - e^{-0.04\sqrt{(15/4)}}} = 1,490.$$

Answer: E

8. Let X and Y be the payments for the first year and the second year respectively. Then the present value of the payments is

$$\begin{aligned} S &= X(1.06)^{-1} + Y(1.06)^{-2} \\ Var(S) &= (1.06)^{-2}Var(X) + (1.06)^{-4}Var(Y) + 2(1.06)^{-3}Covar(X,Y) \\ &= (1.06)^{-2}(1200)^2/12 + (1.06)^{-4}(1200)^2/12 + 2(1.06)^{-3}(10)^5 = 369,775. \\ SD &= \sqrt{369775} = 608. \end{aligned}$$

Answer: B

9. $\mu^a = 0.1\mu^\tau$ and $\mu^n = 0.9\mu^\tau$. Hence the APV of benefit is

$$\{(0.9)(1000) + (0.1)(10000)\}\,\overline{A}_x = 1900\,\overline{A}_x.$$

The APV of the insurance before the accidental death benefit was added is $1000\,\overline{A}_x$. Since the annuity factor will remain the same, the new premium will be $(1.9)(10) = 19$.

Answer: E

10. The APV is that of a 35-year deferred annuity for Peter plus a 25-year deferred annuity for Janice minus a 35-year deferred annuity for the joint status because we would have counted the annuity twice after they have both attained age 65. Thus

$$\begin{aligned} APV &= \int_{35}^{\infty} e^{-0.08t}\,dt + \int_{25}^{\infty} e^{-0.1t}\,dt - \int_{35}^{\infty} e^{-0.12t}\,dt \\ &= (e^{-2.8}/0.08) + (e^{-2.5}/0.1) - (e^{-4.2}/0.12) = 1.46 \end{aligned}$$

Answer: A

11. For given force of mortality μ, the APV of benefit is $\mu/(\mu+\delta)$. Therefore the unconditional expectation of the present value of the benefit is

$$\begin{aligned} APV &= \frac{1,000,000}{\delta}\int_{\delta}^{2\delta} \frac{\mu}{\mu+\delta}\,d\mu \\ &= 1,000,000\int_{1}^{2} \frac{y}{y+1}\,dy \;(\text{ with } \;\mu = y\,\delta) \\ &= 1,000,000\int_{1}^{2} \left(1 - \frac{1}{y+1}\right)\,dy \\ &= 1,000,000\,(1 - \ln(3/2)) = 594,535. \end{aligned}$$

So the relative security loading is $(625,000/594,535) - 1 = 0.0512$.

Answer: A

12. This problem shows an interesting result. The limiting premium is the benefit premium. Let us do this generally. Let $P(n)$ be the premium per policy and L the loss per policy when there are n policies. Then

$$\begin{aligned} E(L) &= \overline{A}_x - P(n)\,\overline{a}_x \\ SD(L) &= \left(1+\frac{P(n)}{\delta}\right)\sqrt{{}^2\overline{A}_x - \overline{A}_x^2} = K\left(1+\frac{P(n)}{\delta}\right). \end{aligned}$$

We have denoted the term under the square root by the positive constant K. The mean of the aggregate loss (call it S) is $nE(L)$ and the standard deviation is $\sqrt{n}\,SD(L)$. Hence in the normal approximation,

$$Pr(S > 0) \leq p$$

is the same as

$$1 - \Phi\left(\frac{-nE(L)}{\sqrt{n}SD(L)}\right) \leq p$$

or

$$\Phi\left(\frac{-\sqrt{n}\left(\overline{A}_x - P(n)\,\overline{a}_x\right)}{K\left(1+\frac{P(n)}{\delta}\right)}\right) \geq 1-p.$$

The argument of Φ is an increasing function of $P(n)$ for fixed n. Therefore, for fixed n, the smallest premium that will assure that the probability of a positive loss is no more than p is given by

$$\frac{-\left(\overline{A}_x - P(n)\,\overline{a}_x\right)}{K\left(1+\frac{P(n)}{\delta}\right)} = \frac{1}{\sqrt{n}}z_{1-p}$$

where z_{1-p} is the $100(1-p)$-th percentile of the standard normal variable. Solving for $P(n)$ we note that for some constants c and d,

$$\lim_{n\to\infty} P(n) = \lim_{n\to\infty}\frac{\overline{A}_x + c/\sqrt{n}}{\overline{a}_x + d/\sqrt{n}} = \overline{A}_x/\overline{a}_x.$$

which is the benefit premium. In this case it equals $\mu = 0.01$.

Answer: B

13. Let $Z = \max(T(x), T(y))$ and $W = \min(T(x) < T(Y))$. Then

$$\begin{aligned} E(Z-W) &= E(Z+W) - 2E(W) = E(T(x)) + E(T(y)) - 2E(W) \\ &= (1/0.02) + (1/0.03) - 2/(0.05) = 43.33. \end{aligned}$$

Answer: C

14. Equating the APVs of benefit and premiums,

$$(20,000)(0.1)(1.06)^{-1} + 12,000(0.9)(0.1)(1.06)^{-2} + (0.1)(2P)(0.81)(1.06)^{-2}$$
$$= P\{1 + (0.9)(1.06)^{-1}\},$$

and solving for P we get $P = 1670.49$. The reserve at the end of the first year is

$$(1.06/0.9)\{1670.49 - 20000(0.1)/1.06\} = -255.$$

Alternatively, you may use recursion.

$$\begin{aligned} 0 + P &= 1.06^{-1}(0.1)(20) + 1.06^{-1}(0.9)\ {}_1V \\ {}_1V + P &= 1.06^{-1}(0.1)(12) + 1.06^{-1}(0.9)(0.2P) \end{aligned}$$

Eliminating P and solving for ${}_1V$ we get -255 for the answer.

Answer A

15. The total time of the grant has a gamma distribution with pdf $f(t) = te^{-t}$. Therefore the APV is

$$242,000 \int_0^\infty e^{-0.1t}(1 - F(t))\, dt = 242,000\overline{a}_0(t)$$
$$= (242,000/0.1)\left\{1 - \overline{A}_0(t)\right\} = 242,000(10)(1 - M_T(-0.1))$$
$$= (242,000)(10)\left\{1 - (1/1.1)^2\right\} = 420,000.$$

Alternatively you can integrate by parts and get

$$\int_0^\infty e^{-0.1t} te^{-t}\, dt = 1/1.21.$$

Answer: D

16. Since with UDD

$$\overset{\circ}{e}_x = \int_0^\infty {}_tp_x\,dt = \int_0^1 {}_tp_x\,dt + \int_1^\infty {}_tp_x\,dt = (1/2) + (1/2)p_x + p_x\,\overset{\circ}{e}_{x+1},$$

I is false.

UDD does not imply linearity of the distribution of the joint status between integer years. Even if we assume independence, for integer ages x and y and $0 < t < 1$, ${}_tp_{xy} = {}_tp_x\,{}_tp_y = (1 - tq_x)(1 - tq_y)$ is quadratic in t. II is false.

The relation III holds only for term. Unless the probability of survival to n is 0, this statement is false.

Answer: D

17. $E(S) = E[(S-1)_+] + 1 - F(0) = 0.867879 + 1 - e^{-1} = 1.5$. Therefore if X is the individual claim, then $E(X) = 1.5 = (1)(p) + 2(1-p)$. This gives $Pr(X = 1) = Pr(X = 2) = 1/2$. Therefore

$$\begin{aligned}
Pr(S = 1) &= Pr(N = 1)Pr(X = 1) = (1/2)e^{-1} \\
F(1) &= (3/2)e^{-\lambda} = 0.551819 \\
E[(S-2)_+] &= E[(S-1)_+] - [1 - F(1)] = 0.419698 \\
E[(S-1.5)_+] &= (1/2)(0.867879 + 0.419698) = 0.6437885 \\
E[(S-1)_+] - E[(S-1.5)_+] &= 0.867879 - 0.6437885 = 0.22409.
\end{aligned}$$

Answer: B

18. Use Hattendorf recursion.

$$Var({}_hL|K(x) \geq h) = Var(\Lambda_h|K(x) > h) + v^2 p_{x+h} Var({}_{h+1}L|K(x) \geq h+1)$$

along with

$$\Lambda_h = \begin{cases} 0 & K(x) = 0, 1, \cdots, h-1 \\ (10 - {}_{h+1}V)\, v p_{x+h} & K(x) = h \\ -(10 - {}_{h+1}V)\, v q_{x+h} & K(x) \geq h+1. \end{cases}$$

$Var({}_4L|K(x) \geq 4) = 0$. We are given that ${}_4V = 5.672$. Since $p_{x+3} = 1/2$, $\Lambda_3 = (10 - 5.672)(1/2)(0.8) = 1.7312$ with probability $1/2$ and -1.7312 with probability $1/2$. Therefore

$$Var(\Lambda_3|K(x) \geq 3) = (1.7312)^2 = 2.997.$$

$$Var({}_3L|K(x) \geq 3) = 2.997.$$

One could also do this without the use of Hattendorf's theorem.

Answer: D

19. For Pareto, with $\alpha > 1$, the median is $m = \theta\,(2^{1/\alpha} - 1) < \theta/\alpha < \theta/(\alpha - 1) = E(X)$. So I is false.

$$LER = \frac{E(X \wedge d)}{E(X)} = 1 - \left(\frac{\theta}{d+\theta}\right)^{\alpha-1}$$

increases with α for $d > 0$. Hence II is true.

The hazard rate is $\alpha/(\theta + x)$ which is a decreasing function of x. Hence III is true.

Answer: B

20. If the deductible is 100, then the expected payment is (200)(0.1) + (300)(0.1)+(500)(0.03)+(700)(0.02) = 79. If the deductible is 300, then the expected payment is (100)(0.1) + (300)(0.03) + (500)(0.02) = 29. If the deductible is increased by 200, the expected payment decreases by 50. We need the deductible such that the expected payment will decrease by 5 from 79. Hence the deductible is 120, and

$$
\begin{aligned}
E(X \wedge 120) &= (0)(0.6) + 100(0.15) + (120)(0.25) = 45 \\
E(X) &= 15 + 30 + 40 + 18 + 16 = 119 \\
LER &= 45/119 = 0.378.
\end{aligned}
$$

Answer: D

21. Since the distribution is exponential, the cost per payment has the same distribution as the loss. If Y is the loss next year, then $Pr(Y \leq 80) = 1 - e^{-80/110} = 0.517$.

Answer: C

22. Use the double expectation formula:

$$
\begin{aligned}
E(X|\theta) &= \theta/2,\ E(X^2|\theta) = 2\theta^2/2 = \theta^2 \\
E(X) &= E(E(X|\theta)) = (1/2)(0.01)\int_{100}^{200} \theta \, d\theta = (1/4)(0.01)\left(200^2 - 100^2\right) = 75 \\
E(X^2) &= (0.01)\int_{100}^{200} \theta^2 \, d\theta = (0.01)(1/3)\left(200^3 - 100^3\right) = 23,333 \\
Var(X) &= 23,333 - 75^2 = 17,708.
\end{aligned}
$$

Answer: D

23.

$$\begin{aligned}
5/48 = p_4/(1-p_0) &= \left(a+\frac{b}{4}\right)p_3/(1-p_0) = \left(a+\frac{b}{4}\right)(1/6) \\
\left(a+\frac{b}{4}\right) &= 5/8 \qquad (1) \\
1/6 = p_3/(1-p_0) &= \left(a+\frac{b}{3}\right)p_2/(1-p_0) = \left(a+\frac{b}{3}\right)(1/4) \\
\left(a+\frac{b}{3}\right) &= 2/3 \qquad (2)
\end{aligned}$$

Solving Eqs.(1) and (2) we get $a = 1/2,\ b = 1/2$. Now

$$\begin{aligned}
1/4 = p_2/(1-p_0) &= (1/2)(1+1/2)Pr(N=1|N>0);\ Pr(N=1|N>0) = 1/3 \\
Pr(N=1) &= (1/2)(1+1/1)Pr(N=0) = Pr(N=0) \\
Pr(N=1|N>0) &= \frac{Pr(N=0)}{1-Pr(N=0)} = 1/3 \\
Pr(N=0) &= 1/4 = Pr(N=1) \\
Pr(N>1) &= 1-(1/4)-(1/4) = 1/2.
\end{aligned}$$

Answer: A

24. The number of "failures" before the third success has a negative binomial distribution with $\beta = 9$ and $r = 3$. Of each of these "failures" the probability that it is of a certain type (topology) is 1/3. Hence the number of topology questions before the third Exam-M question has a negative binomial distribution with parameters $\beta' = (9)(1/3) = 3$ and $r = 3$; and the probability that there will be no topology questions before the third Exam-M question is $(1+\beta')^{-r} = 4^{-3} = 1/64$. The probability that there will be at least one topology question is 63/64.

Answer: E

25. Let S be the number of claims. Then S is compound Poisson with $\lambda = (3/4)(2)$ and secondary distribution, $Pr(X = 1) = Pr(X = 2) = Pr(X = 3) = 1/3$. Therefore

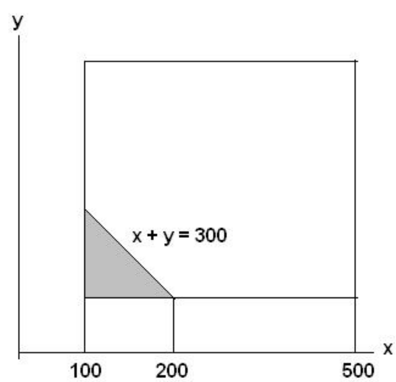
$$\begin{aligned} Pr(S = 0) &= e^{-1.5};\ Pr(S = 1) + Pr(N = 1)Pr(X = 1) = (1.5)e^{-1.5}(1/3) = 0.5e^{-1.5} \\ Pr(S \geq 2) &= 1 - Pr(S = 0) - Pr(S = 1) = 1 - 1.5\,e^{-1.5} = 0.6653. \end{aligned}$$

Answer: E

26. If there are no claims, the dividend is $(0.1)(300) = 30$. If there is one claim, the expected dividend is

$$E(D|N = 1) = (0.1)(1/400)\int_{100}^{300}(300-x)\,dx = (0.1)(1/800)(200)^2 = 5.$$

If there are two claims, X and Y, then the shaded region in the figure below is where the payment is $300 - X - Y$.



$$\begin{aligned} E(D|N = 2) &= (0.1)(1/400)^2\int_{100}^{200}\int_{100}^{300-x}(300 - x - y)\,dy\,dx \\ &= (0.1)(1/400)^2(1/2)\int_{100}^{200}(200 - x)^2\,dx \\ &= (0.1)(1/400)^2(1/2)(1/3)(100)^3 = 0.1042. \\ E(D) &= (1/3)\{30 + 5 + 0.1042\} = 11.701. \end{aligned}$$

Answer: C

27. Let G be the expense-loaded premium. Then

$$G\,\ddot{a}_{40} = 1,000\,A_{40} + 0.05G\,\ddot{a}_{40:\overline{10|}} + 0.25G + 0.02G\ddot{a}_{40} + 4\ddot{a}_{40} + 6.$$

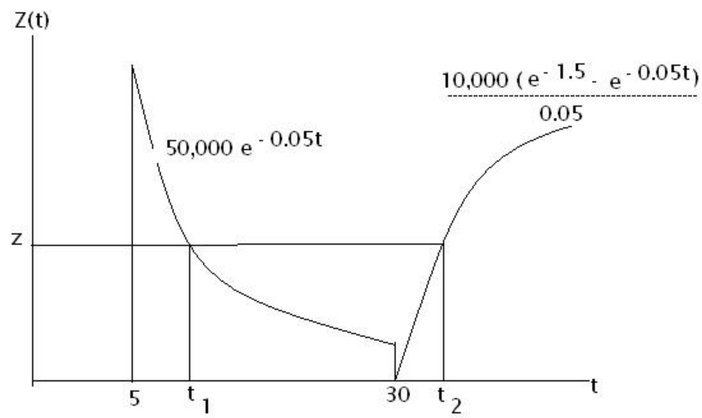
or

$$G = \frac{1,000\,A_{40} + 4\ddot{a}_{40} + 6}{0.93\,\ddot{a}_{40} + 0.05\,{}_{10}E_{40}\ddot{a}_{50} - 0.25}.$$

Using the ILT we get $G = 16.32$.

Answer: E

28. The following figure shows a graph of the present value of the payment, $Z(t)$ as a function of the time death. We need z such that



$Pr(Z \leq z) = 0.9$. Assume that z is as shown. Calculate z and verify that it is less than $Z(5)$.

Now

$$\begin{aligned} Pr(Z \le z) &= Pr(T < 5) + Pr(t_1 < T \le t_2) \\ &= Pr(T < 5) + Pr(T > t_1) - Pr(T > t_2) \\ &= 1 - e^{-0.25} + e^{-0.05t_1} - e^{-0.05t_2}. \end{aligned} \tag{3}$$

From the Figure note that

$$50,000e^{-0.05t_1} = (10,000/0.05)(e^{-1.5} - e^{-0.05t_2}) = z$$

or

$$e^{-0.05t_1} = z/50,000; \quad \text{and} \quad e^{-0.05t_2} = e^{-1.5} - 0.05z/10,000.$$

Using these in Eq.(1) and equating it to 0.9 we get

$$Pr(Z \le z) = 1 - e^{-0.25} + z/50,000 - e^{-1.5} + 0.05z/10,000 = 0.9$$

or $z = 36,077$. $Z(5) = 50,000e^{-0.25} = 38940 > 36,077$. So we are okay.

Answer: A

29. Since the probability that a claim is less than 100 is 0.1, by the decomposition property of Poisson, M is Poisson with mean $0.9(4) = 3.6$ and N is Poisson with mean $0.1(4) = 0.4$ and - this is very important - M and N are independent. Therefore $E\{M/(N+1)\} = E(M)E\{(N+1)^{-1}\}$. Now

$$\begin{aligned} E\{(N+1)^{-1}\} &= \sum_{n=0}^{\infty} \frac{0.4^n}{(n+1)!} e^{-0.4} \\ &= (1/0.4) \sum_{n=0}^{\infty} \frac{0.4^{n+1}}{(n+1)!} e^{-0.4} \\ &= (1/0.4)e^{-0.4}(e^{0.4} - 1) = (1/0.4)(1 - e^{-0.4}) \end{aligned}$$

and since $E(M) = 3.6$,

$$E\{M/(N+1)\} = 3.6(1/0.4)(1 - e^{-0.4}) = 2.967.$$

Answer: C

30. Let T stand for the future lifetime of the newborn. Then for $x \leq \omega$,

$$\begin{aligned}
Z(x) &= T \text{ if } T \leq x \text{ and } x \text{ if } T > x = T \wedge x \\
W(x) &= 0 \text{ if } T \leq x \text{ and } T - x \text{ if } T > x = (T - x)_+ \\
Z(x) + W(x) &= T \\
Z(x)W(x) &= xW(x) \\
E(Z(x)) &= \overset{\circ}{e}_{0:\overline{x}|} = \int_0^x {}_tp_x \, dt = \frac{2\omega x - x^2}{2\omega} \\
E(W(x)) &= E(T) - E(Z(x)) = (\omega - x)^2/2\omega \\
Cov(Z(x), W(x)) &= xE(W(x)) - E(Z(x))E(W(X)) \\
&= \{x - E(Z(x))\} E(W(x)) = (1/4\omega^2)x^2(\omega - x)^2.
\end{aligned}$$

By symmetry, the maximum occurs at $x = \omega/2$ and the corresponding covariance is $\omega^2/64$. With $\omega = 100$, this gives 156.25. Note that if $x > \omega$, $E(W(x)) = 0$.

Answer: A

31. The payment is

$$\begin{aligned}
(X - d)_+ + 0.25(d - X)_+ &= X - X \wedge d + 0.25\,(d - X \wedge d) \\
&= X + 0.25d - 1.25X \wedge d \\
E\left((X - d)_+ + 0.25(d - X)_+\right) &= E(X) + 0.25d - 1.25 \int_0^d \{1 - F(x)\} \, dx \\
\frac{d}{dd} E\left((X - d)_+ + 0.25(d - X)_+\right) &= 0.25 - 1.25 + 1.25F(d) = 0
\end{aligned}$$

when $F(d) = 1/1.25 = 0.8$. Since the second derivative is $1.25f(d) > 0$, this is the minimum or $F(d_m) = 0.8$. The expected number of losses that exceed d_m is $100\{1 - F(d_m)\} = 20$.

Answer: B

32. I is false because both pdfs behave like x^{-2} for large x.

II is true only if A is an integer.

III is true, because, the force of mortality decreases with age and so ${}_tp_x$ is an increasing function of x.

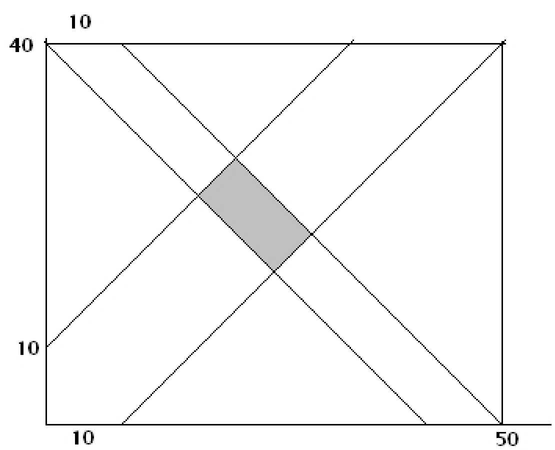
Hence III is the only true statement.

Answer: D

33. If p is the 90-th percentile of $\min(T(x), T(y), T(z))$, then $e^{-0.3p} = 0.1$. If $W = \max(T(x), T(y), T(z))$, then $F_W(p) = (1 - e^{-0.1p})^3 = \left\{1 - (0.1)^{1/3}\right\}^3 = 0.154$

Answer: C

34. The joint pdf of the future lifetimes equals 1/2000 inside the rectangle shown below and 0 elsewhere. The event that X and Y will die within



10 years of each other and the average ages at death is between 75 and 80 corresponds to the shaded rectangle with sides $10\sqrt{2}$ and $10/\sqrt{2}$. Its area is 100. Therefore the desired probability is $100/2000 = 0.05$.

Answer: B

35. Let X be the number of hours that Jean will study in one day and S the total number of hours she will study in 80 days. Let H represent heads in a toss. Then

$$\begin{aligned}
E(X|H) &= 7/2 \\
E(X^2|H) &= 91/6 \\
E(X) &= 7/4 \\
E(X^2) &= 91/12 \\
Var(X) &= (91/12) - (49/16) = 217/48 \\
E(S) &= (7/4)(80) = 140 \\
Var(S) &= (217/48)(80) = 361.7 \\
Pr(S \geq 120) &= 1 - Pr(S \leq 119) = 1 - \Phi\left(\frac{119-140}{\sqrt{361.7}}\right) = \Phi(1.104).
\end{aligned}$$

Answer: C

36. The Poisson process has independent increments. So, $S(1)$, $S(2)-S(1)$ and $S(3)-S(2)$ are independent. Since the process is stationary, $S(2)-S(1)$ and $S(3)-S(2)$ have the same distribution as $S(1)$. Therefore

$$\begin{aligned}
A &= (1/3)\,[3S(1) + 2\{S(2) - S(1)\} + S(3) - S(2)] \\
Var(A) &= (1/9)\,\{(9)Var(S(1)) + 4Var(S(1)) + Var(S(1))\} \\
&= (14/9)(1)E(X^2) = (14/9)(18) = 28.
\end{aligned}$$

Answer: C

37. Given that the number of claims is n, the probability that the largest claim is less than 1000 is the same as the probability that every one of the n claims is less than 1,000. The probability that a claim is less than 1,000 is $1-(1/2)^2 = 3/4$. Therefore the probability that the maximum claim is less than 1,000 is

$$\sum_{n=0}^{\infty}(1/3)(2/3)^n(3/4)^n = (1/3)\sum_{n=0}^{\infty}(1/2)^n = 2/3.$$

Answer: C

38. If (x) dies first the expected time between deaths is $1/0.06$ and if (y) dies first, the expected time between deaths is $1/0.02$.The probability that (x) will die first is $0.02/0.08$ and the probability that (y) will die first is $0.06/0.08$. Therefore the expected time between deaths is

$$(1/4)(1/0.06) + (3/4)(1/0.02) = 41.7.$$

Answer: D

39. Count money in thousands. Denote by X the loss in the old model and by Y the loss in the new model. From the Table, $E(X) = 10$, $E(X \wedge 30) = 7.5$ and $F(30) = 15/16$. Denote the PDF in the old model by f and that in the new model by g. Then

$$g(y) = c(y+10)^{-3} = \frac{c}{200}f(y),\ 0 < y < 30$$

and 0 otherwise.

$$\frac{c}{200}\int_0^{30} f(y)\,dy = \frac{c}{200}F(30) = 1.$$

This gives $c = 640/3$.

In the new model,

$$E(Y) = \frac{c}{200}\left[E(X \wedge 30) - 30(1-F(30))\right] = (16/15)[7.5 - (30/16)] = 6.$$

The decrease in the premium is $(1.5)(10-6) = 6$. This is in thousands.

Answer: E

40. Let $\mathbf{P}_n$ be the probability vector at the beginning of year n, $n = 0, 1, 2, 3$. Then

$$\mathbf{P}_0 = [1 \;\; 0 \;\; 0];\; \mathbf{P}_1 = [0.8 \;\; 0.1 \;\; 0.1]$$
$$\mathbf{P}_2 = \mathbf{P}_1\mathbf{Q}_1 = [0.61 \;\; 0.14 \;\; 0.25];\; \mathbf{P}_3 = \mathbf{P}_2\mathbf{Q}_2 = [0.422 \;\; 0.136;\; 0.442].$$

To calculate the APV of benefits, note that persons in state 2 are paid a benefit each year, whereas the death benefit is paid only one time. So the APV of benefits is

$$[(100)(0.1) + (1,000)(0.1)](0.8) + [(0.14)(100) + (0.25 - 0.10)(1,000)](0.8)^2$$
$$+(0.442 - 0.25)(1,000)(0.8)^3 = 291.264.$$

The APV of premiums is

$$P[1 + (0.8)(0.8) + ((0.61)(0.8)^2] = 2.0304P.$$

Therefore $P = 291.264/2.0304 = 143.45$.

Answer: E